Test for the fulfillment of the Additional Learning Requirements
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A

(1) The set of solutions of the inequality $\dfrac{3x-2}{x+4} > 1$ is:

A $]-\infty, -4[\cup]3, +\infty[$

B $]-\infty, -1[$

C $]-\infty, -4[$

D $]3, +\infty[$

E $]1, +\infty[$

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(2) Which of the following statements is true for each $a, b \in \mathbf{R}$?

A $\sqrt{a^2b^2} = \sqrt{a^2}\,\sqrt{b^2}$

B $\sqrt{a^2b^2} = ab$

C $\sqrt{a^2+b^2} = \sqrt{a^2} + \sqrt{b^2}$

D $\sqrt{a^2b^2} = \sqrt{ab}\,\sqrt{ab}$

E $\sqrt{a^2+b^2} = \sqrt{ab} + \sqrt{ab}$

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(3) Which of the following properties is valid for all rhombuses, but *not* for all rectangles?

A The diagonals are congruent.

B The consecutive sides are perpendicular.

C The diagonals bisect.

D The diagonals are perpendicular.

E The opposite sides are parallel.

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(4) If $\sin\alpha = \dfrac{4}{5}$ and $\dfrac{\pi}{2} < \alpha < \pi$, which of the following statements is true?

A $\cos\alpha = \dfrac{3}{5}$

B $\cos\alpha = -\dfrac{3}{5}$

C $\tan\alpha = \dfrac{4}{3}$

D $\cos\alpha = -\dfrac{2}{5}$

E $\tan\alpha = \dfrac{3}{4}$

(5) If $a = \sin(1)$ then:

A $\cos^2(a) + \sin^2(1) = 1$

B $0 < a < 1$

C $a < 0$

D a can take on infinite values

E $a = \frac{\pi}{2}$

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(6) A right triangle is inscribed in a circle of radius 5. Then necessarily:

A the height relative to the hypotenuse cannot be 5

B the perimeter of the triangle is equal to 20

C the sum of the lengths of the catheti is equal to 10

D the hypotenuse has length 10

E one cathetus has length 5

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(7) The polynomial $P_1(x)$ has degree 8 and the polynomial $P_2(x)$ has degree 3. Let $Q(x)$ be the quotient and $R(x)$ be the remainder of the division of $P_1(x)$ by $P_2(x)$. Then

A the degree of $Q(x)$ is 3 and the degree of $R(x)$ is less than 5

B the degree of $Q(x)$ is 5 and the degree of $R(x)$ is 2

C the degree of $Q(x)$ is 3 and the degree of $R(x)$ is less than or equal to 2

D the degree of $Q(x)$ is 5 and the degree of $R(x)$ is less than 3

E the degree of $Q(x)$ is less than or equal to 5 and the degree of $R(x)$ is less than or equal to 3

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(8) For what value of the real parameter a the straight line of equation $(a+3)x+y-2=0$ is parallel to the straight line of equation $y = 2x - 7$?

A $a = -4$

B $a = 0$

C $a = -1$

D $a = -5$

E $a = -10$

(9) Let $f(x) = \log(x^2 + 2x + 1)$. Then the natural domain of f is:

A $[0, +\infty[$

B $]0, +\infty[$

C $]-1, +\infty[$

D $\mathbf{R} \setminus \{-1\}$

E $\mathbf{R}$

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(10) The set of solutions of the inequality $x^3 + 9x^2 \leq 0$ is:

A $[-9, +\infty[$

B $]-\infty, -9] \cup \{0\}$

C $]-\infty, 0]$

D $]-\infty, -9]$

E $[0, 9]$

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(11) Consider the circumferences of equations $x^2 + y^2 = 1$ and $(x-1)^2 + (y-1)^2 - 4 = 0$; then:

A they are concentric

B they have no points in common

C they have the same radius

D they intersect in two points

E one of the two does not intersect the x -axis

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(12) The set of solutions of the inequality $(x+1)(x^2+2)(x^3-3) < 0$ is

A $]1, +\infty[$

B $]-\infty, -1[\cup \left]\sqrt[3]{3}, +\infty\right[$

C $\left]\sqrt[3]{3}, +\infty\right[$

D $\emptyset$

E $\left]-1, \sqrt[3]{3}\right[$

(13) Let $f: \mathbf{R} \to \mathbf{R}$, $f(x) = 5^x$. Then $f(\alpha+1) - f(\alpha)$ is equal to:

A $4 \cdot 5^\alpha$

B 5

C $5 \cdot 5^\alpha$

D 5^α

E 1

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(14) The equation of the straight line parallel to the straight line of equation $x = y$ and passing through the point $(-1, -4)$ is:

A $4x - y = 0$

B $x - y + 3 = 0$

C $4x - y + 3 = 0$

D $x + y + 5 = 0$

E $x - y - 3 = 0$

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(15) If x and y are real numbers, what is the product of 2^{x^2} and 2^{y^2} ?

A 2^{2xy}

B $2^{x^2+y^2}$

C $2^{x^2y^2}$

D $4^{(xy)^2}$

E $4^{x^2+y^2}$

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(16) In the field of real numbers, the equation $3x^4 - 2x^2 - 1 = 0$

A has exactly four solutions

B has exactly three solutions

C has at least four solutions

D has exactly two solutions

E has no solution

(17) Let

$$f : \mathbf{R} \to \mathbf{R}, \qquad f(x) = \frac{2x}{x^2+1}.$$

If $\alpha \in \mathbf{R}$, then $f(3\alpha)$ is equal to

A $\dfrac{3\alpha}{9\alpha^2+1}$

B $\dfrac{6\alpha}{9\alpha^2+3}$

C $\dfrac{2\alpha}{\alpha^2+1}$

D $\dfrac{6\alpha}{9\alpha^2+1}$

E $\dfrac{6\alpha}{3\alpha^2+1}$

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(18) In a right triangle the ratio between a cathetus and the hypotenuse is $\frac{5}{13}$ and the other cathetus is long 48 cm. How long is the perimeter of the triangle?

A 100 cm

B 68 cm

C 115 cm

D 72 cm

E 120 cm

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(19) In the field of real numbers, the equation $\sqrt{x-1} = -(x-3)$

A has the solution 0

B has exactly one solution

C has no solution

D has exactly two solutions

E has exactly three solutions

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(20) Consider the straight lines of equations $2x + y - 2 = 0$ and $3x - y - 3 = 0$; then

A they intersect in the point $(0, 2)$

B they intersect in the point $(0, 1)$

C they intersect in the point $(1, 0)$

D they are parallels

E they coincide