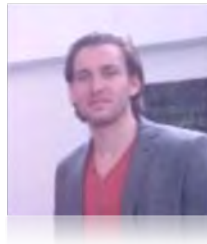


An overview on current methods for interplanetary trajectory optimization

Dario Izzo

Scientific coordinator @ Advanced Concepts Team

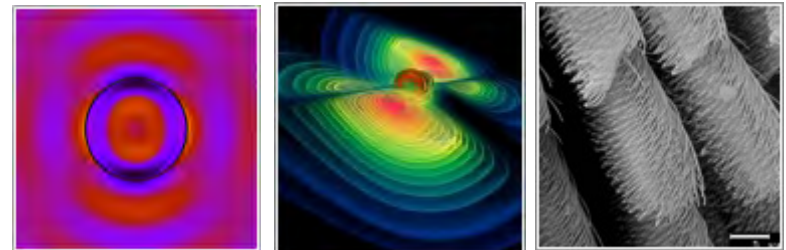
[dario.izzo "at" esa.int](mailto:dario.izzo@esa.int)



monitor, perform and foster research on advanced space systems,
innovative concepts and working methods



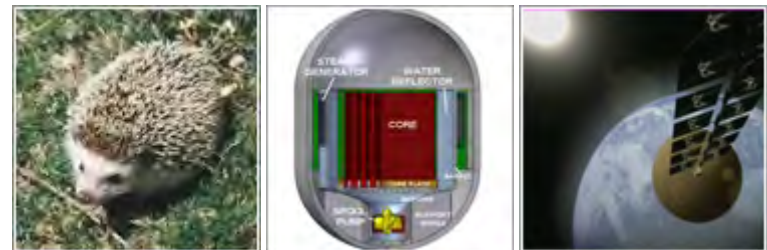
scientific domains with **weak links to space**



too **immature** for regular ESA programmes or projects

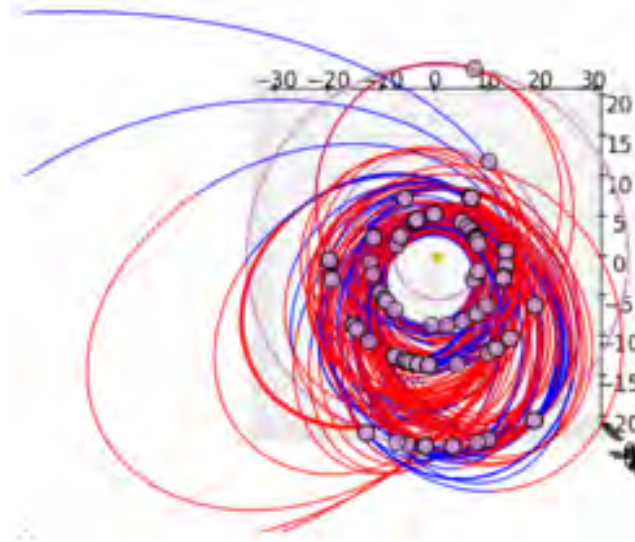


techniques developed in **basic scientific research**



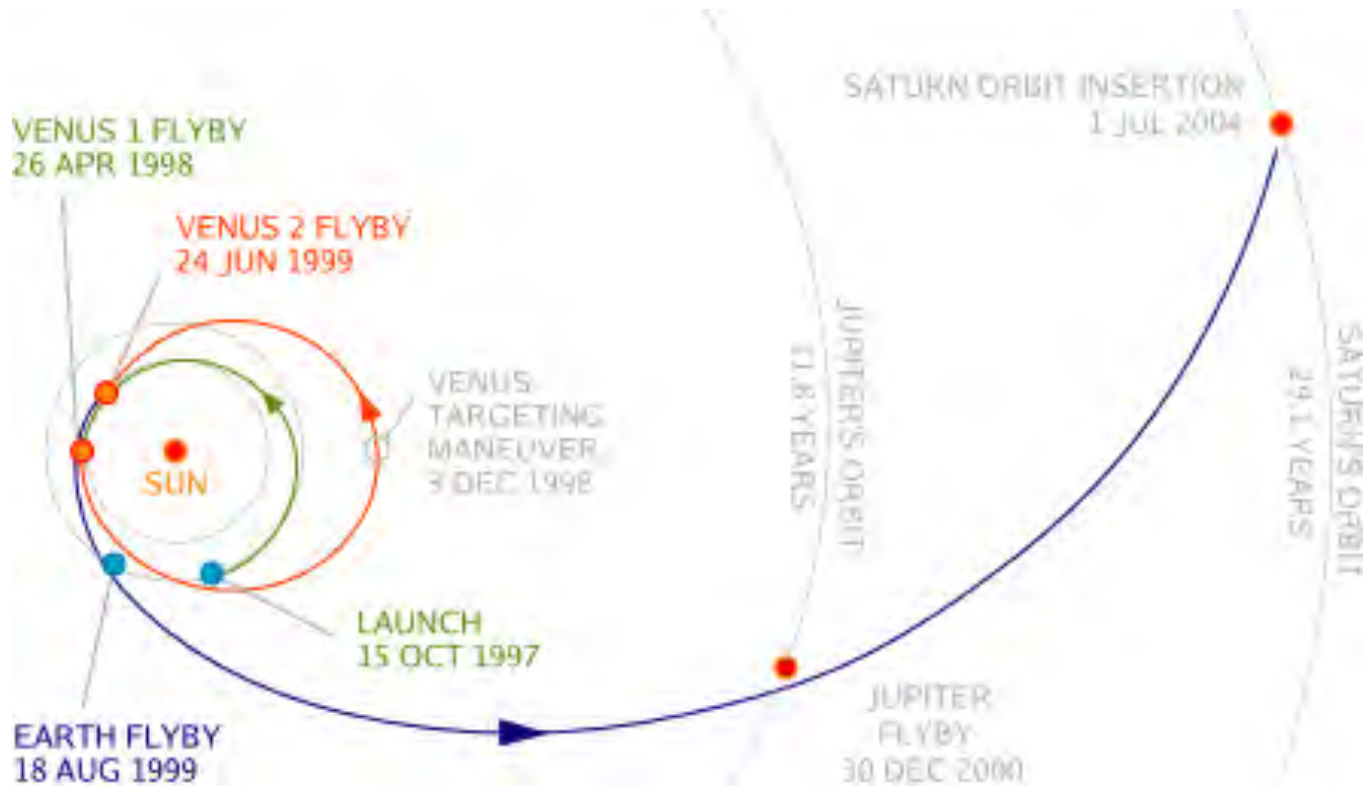
topics on which ESA is **expected to have a position**

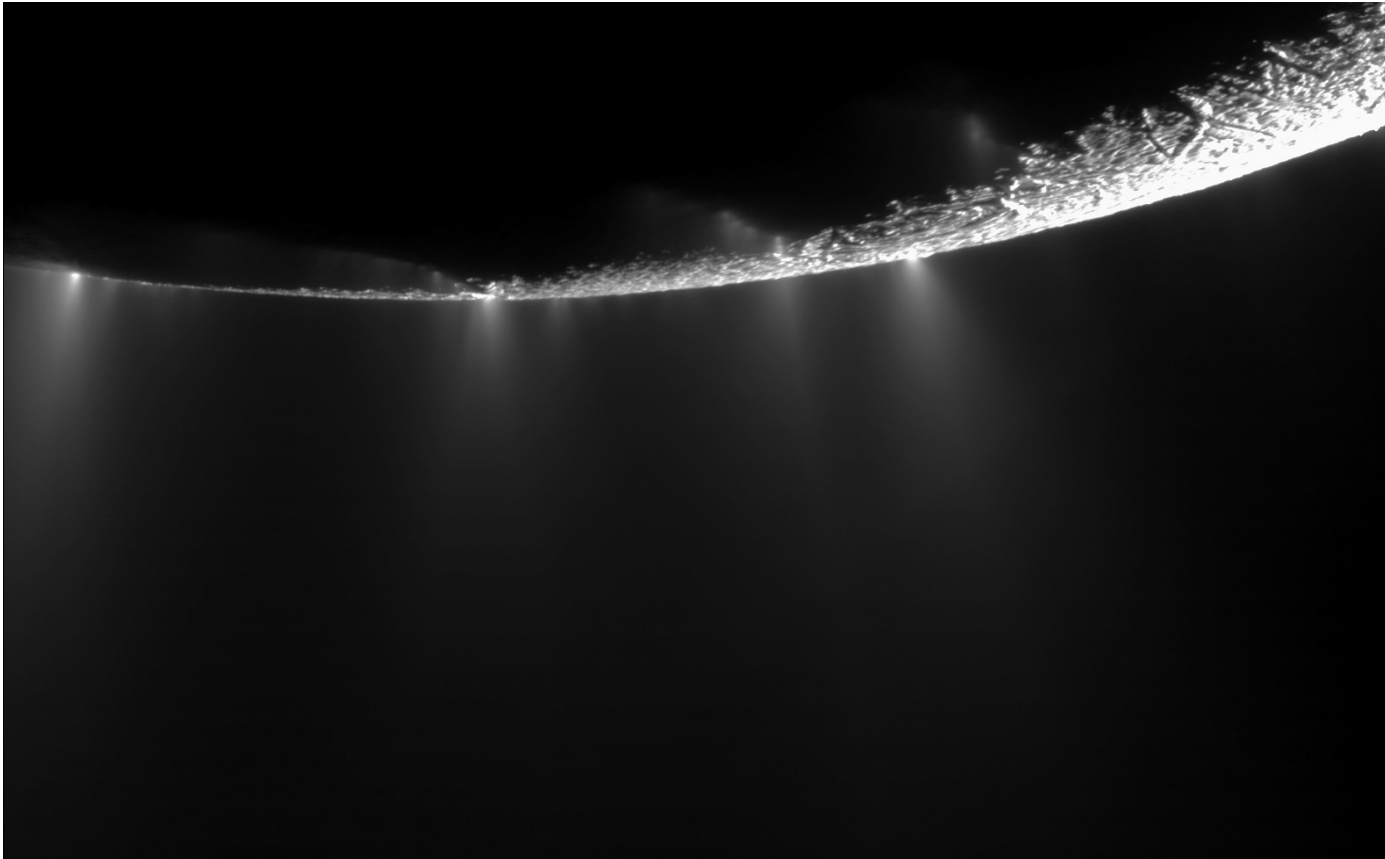
1. Interplanetary Trajectories
 - a. Real Missions
 - b. The GTOC problems
2. Direct vs. Indirect method
 - a. The Optimal Control Problem
 - b. Pontryagin Maximum Principle (indirect methods)
 - c. Transcription Methods (direct methods)
3. A deeper look into direct methods
 - a. Building Blocks (Lambert, Lagrange, Mivovitch)
 - b. Chemical propulsion
 - c. Low-thrust propulsion
4. Web resources
5. Examples: asteroid deflection, human missions to asteroids



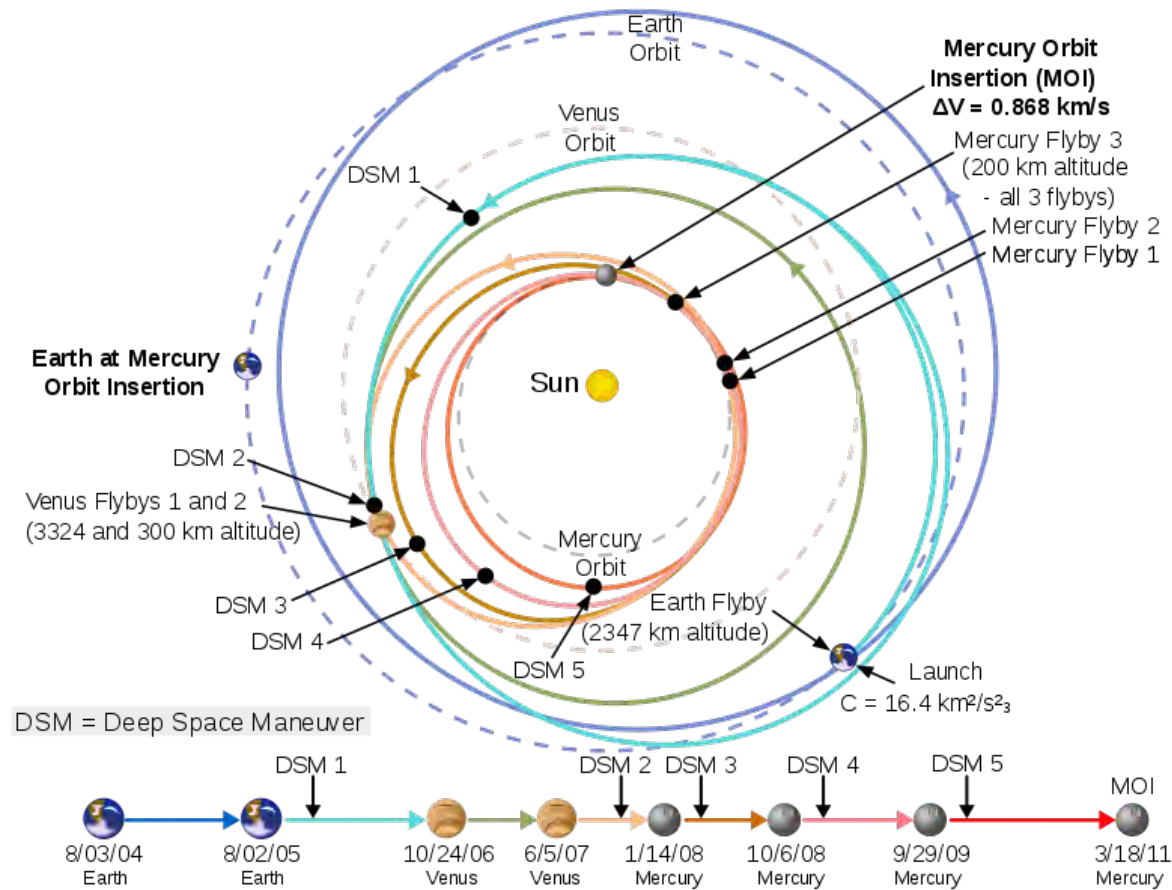
Interplanetary Trajectories

a. Real Missions



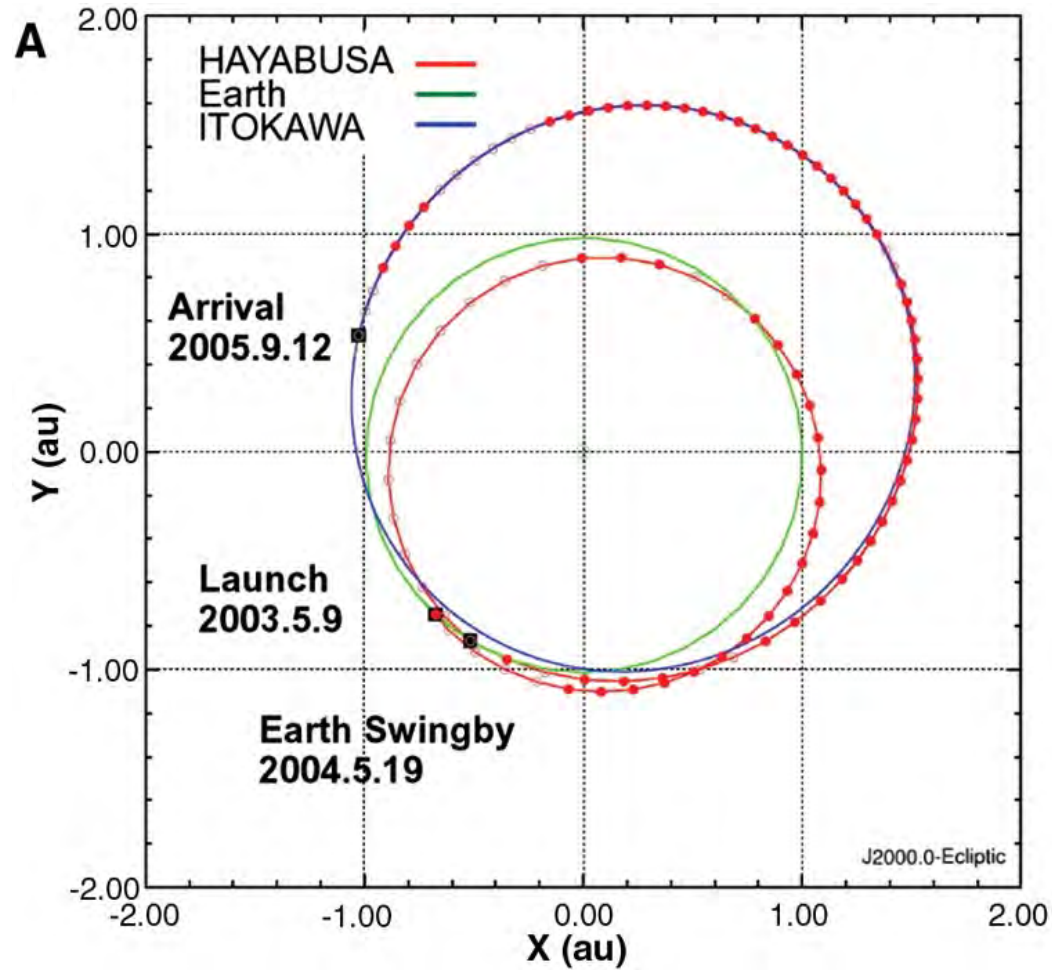


Water in Enceladus?



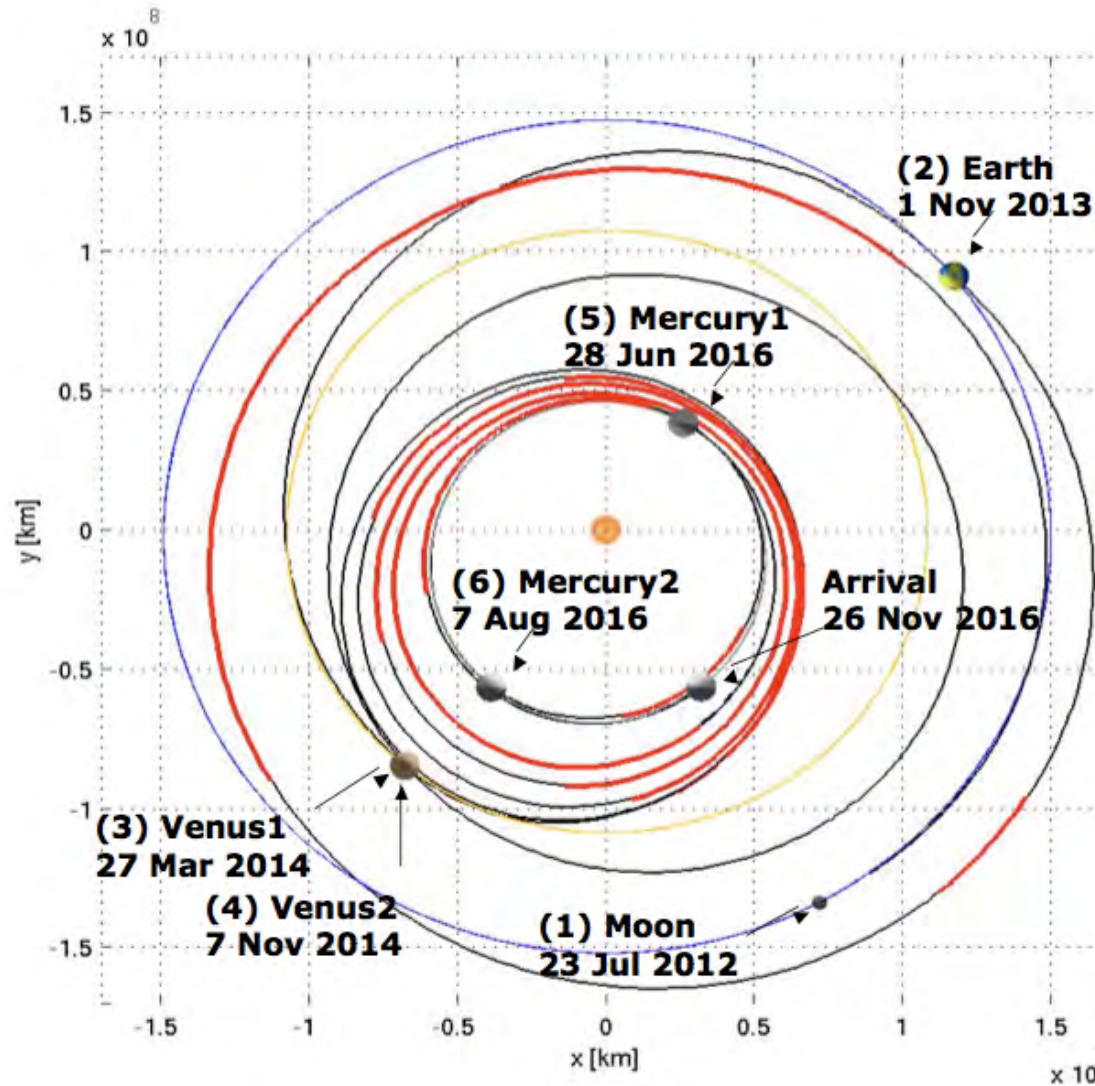


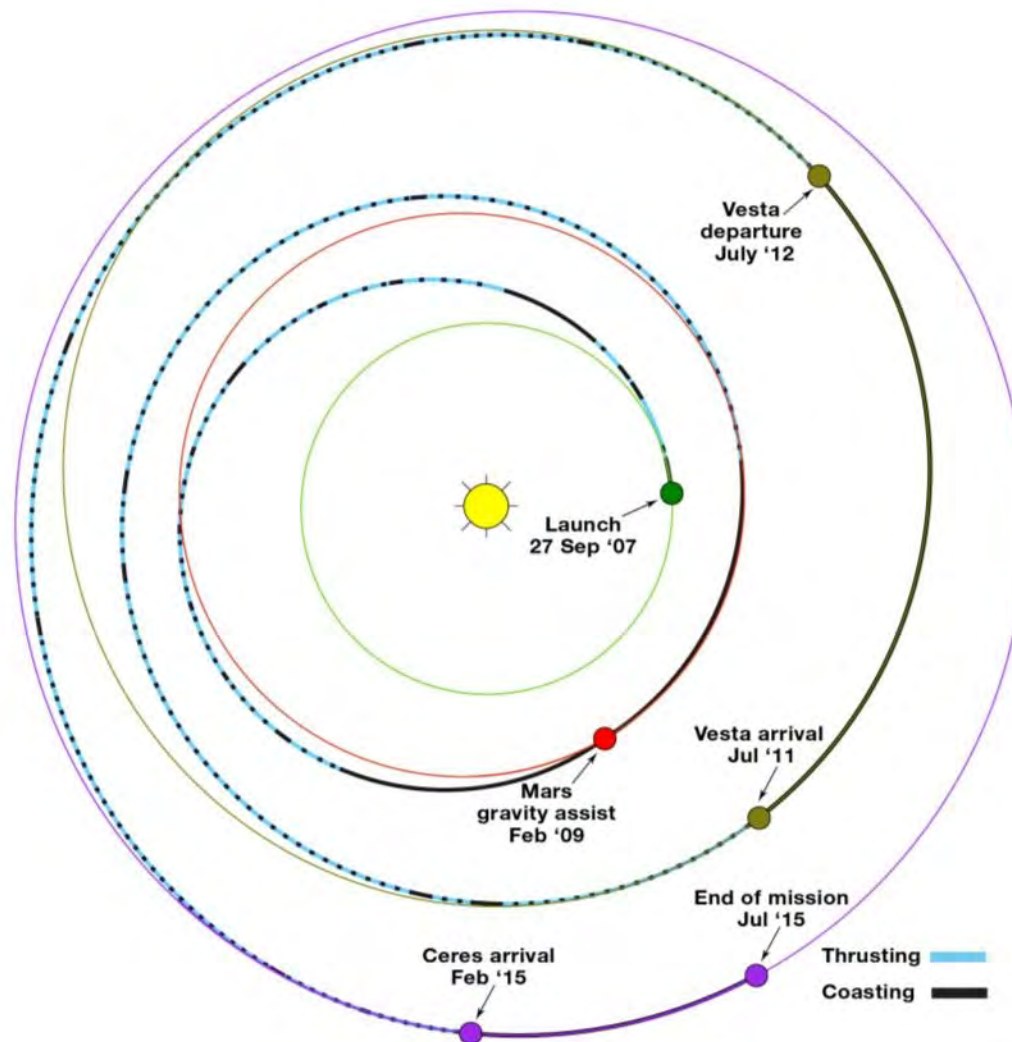
100% Mercury





Rubble piles exist!







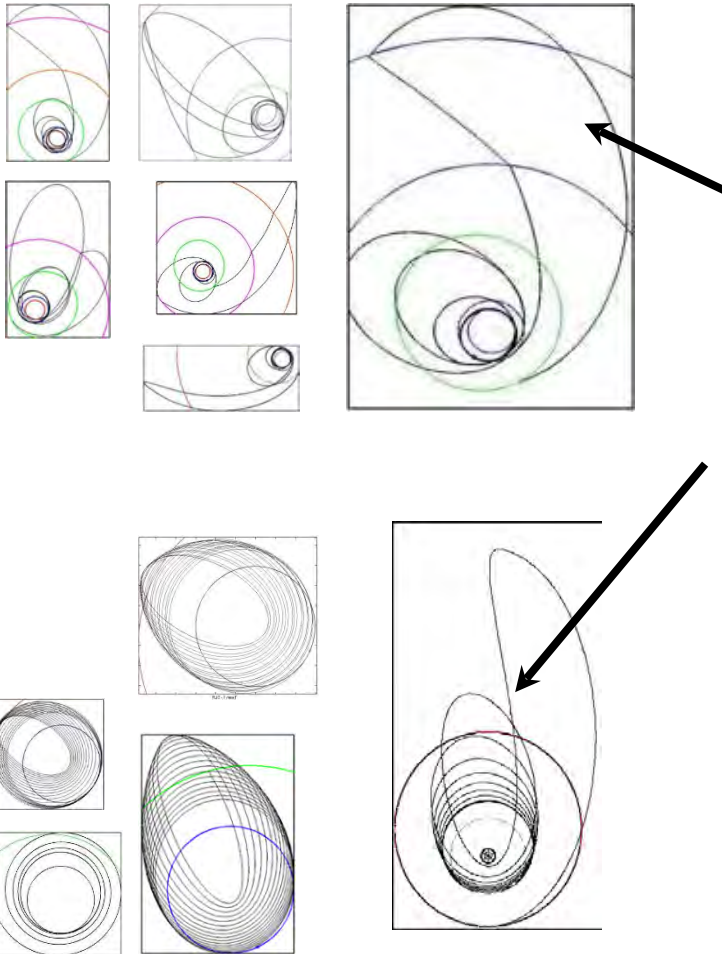
Interplanetary Trajectories

b. GTOC problems

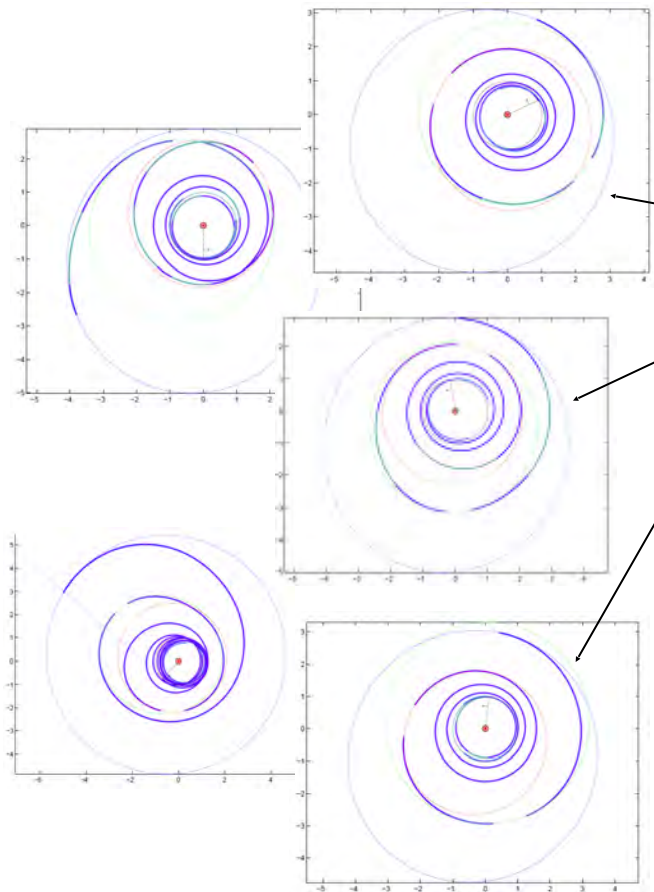
1. GTOC: Global Trajectory Optimization Competition
2. Taking place every year (roughly)
3. Near-to-impossible interplanetary trajectory problem: complexity ensures a clear competition winner
4. Open to academia, industry and space agencies
5. Winners organize and define the following edition
6. Creating a formidable database of challenging problems and solution methods
7. Competition duration is, usually, one month
8. The problem is rigorously defined so that solutions can be ranked with respect to a quantitative objective value
9. Web resources



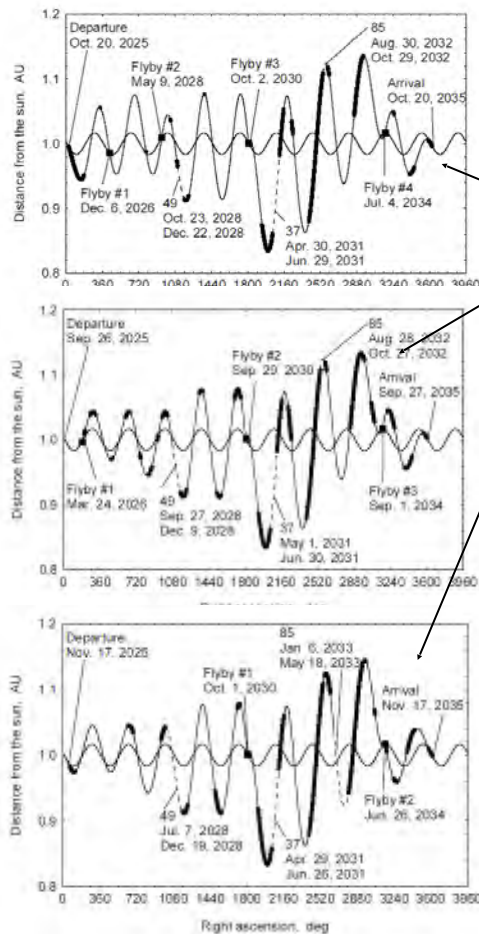
The America's cup of rocket science



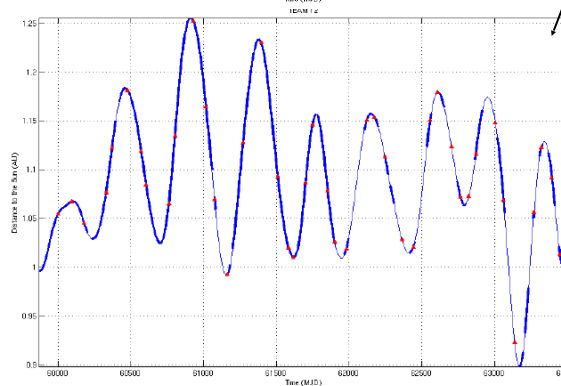
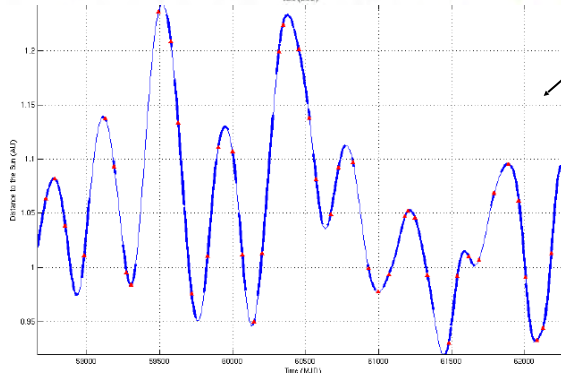
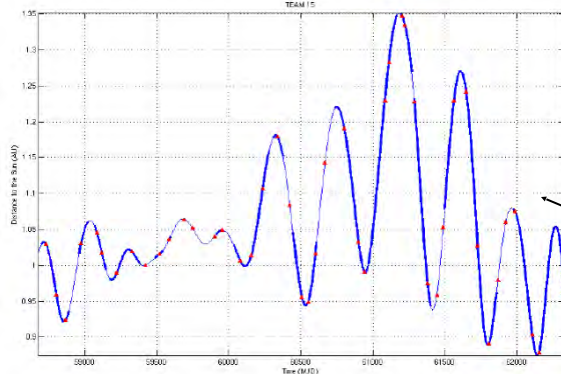
	Team name	Value
1.	Jet Propulsion Laboratory	1,850,000
2.	Deimos Space	1,820,000
3.	GMV	1,455,000
	Moscow Aviation Institute	1,364,000
	Politecnico di Torino	1,290,000
	CNES/CS	1,194,000
	Glasgow University	385,000
	Moscow University	351,152
	Alcatel	330,385
	DLR	330,000
	Tsinghua University	89,000



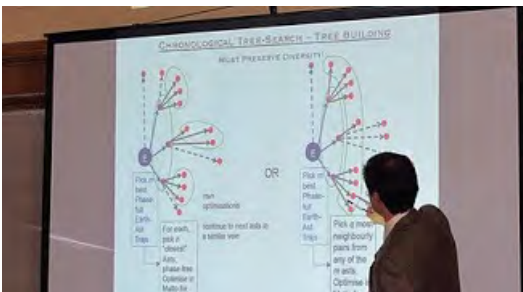
	Team name	Value
1.	Politecnico di Torino	98.64
2.	Moscow Aviation Institute and Khrunichev State Research	87.93
3.	<i>Advanced Concepts Team (ESA)</i>	87.05
	Centre National d'Etudes Spatiales (CNES)	85.43
	GMV Aerospace and Defence	85.28
	German Aerospace Center (DLR)	84.48
	Politecnico di Milano	82.48
	Alcatel Alenia Space	76.37
	Moscow State University	75.08
	Tsinghua University	56.87
	Carnegie Mellon University	27.94



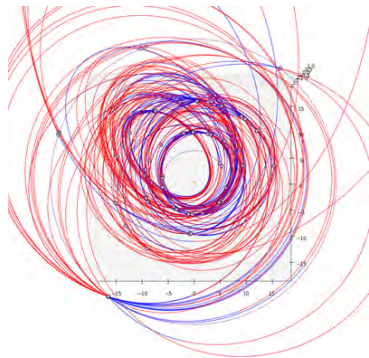
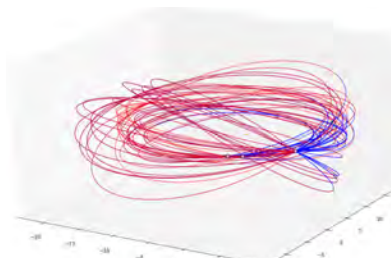
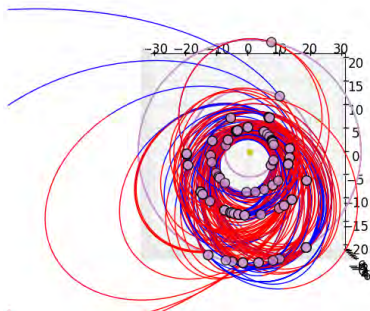
	Team name	Value
1.	Centre National d'Etudes Spatiales	1733
2.	Jet Propulsion Laboratory	1730
3.	Georgia Tech	1721
	Deimos Space	1717
	The Aerospace Corporation	1647
	Thales Alenia Space	1647
	Moscow Aviation Institute and Khrunichev State Research	1658
	GMV Aerospace and Defence	1649
	Moscow State University	1633
	Glasgow University et al.	1606
	Tsinghua University	1565



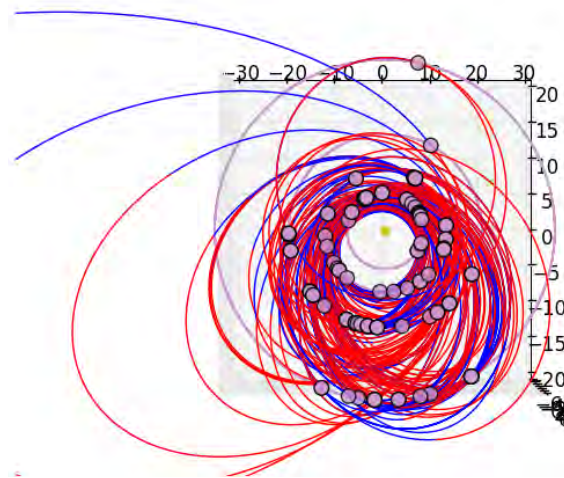
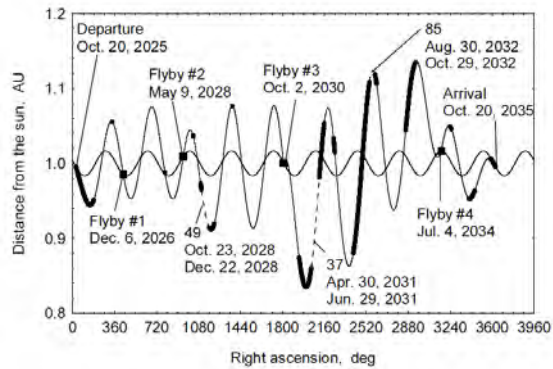
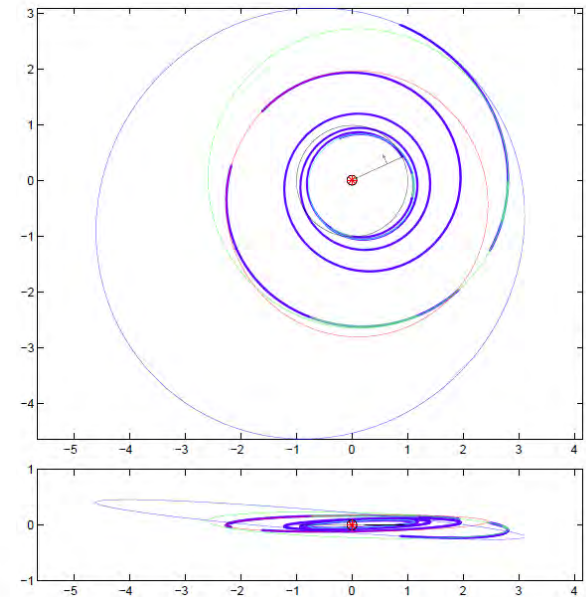
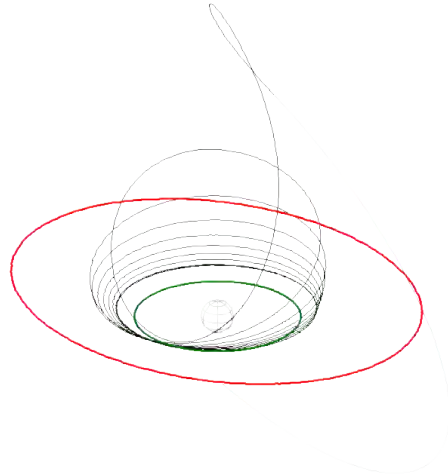
	Team name	Value
1.	Moscow State University	44, 553.46
2.	The Aerospace Corporation	44, 516.83
3.	<i>Advanced Concepts Team (ESA)</i>	42
	Deimos Space	39, 605.44
	GMV Aerospace and Defence	39, 516.30
	Jet Propulsion Laboratory	38
	University of Texas at Austin	36
	University of Glasgow & University of Strathclyde	32
	Thales Alenia Space	29
	University of Trento	27
	University of Bremen, Politecnico Milano	26

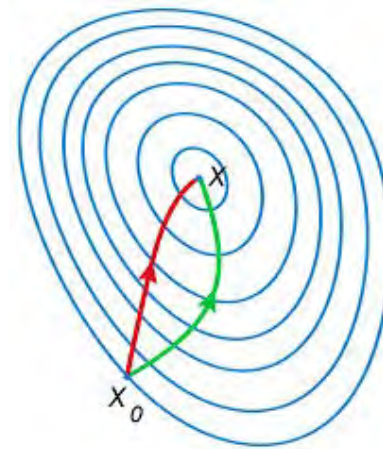
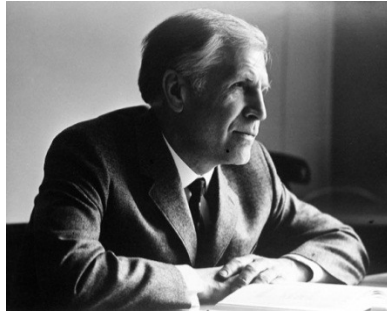


	Team name	Value
1.	Jet Propulsion Laboratory	18, 5459
2.	Politecnico di Torino, Universita' di Roma	17, 5201
3.	<i>Tsinghua University, Beijing</i>	<i>17, 5277</i>
	Advanced Concepts Team (ESA)	16, 5181
	Georgia institute of Technology	16, 5420
	The University of Texas at Austin	15, 5394
	DLR, Institute of Space Systems	14, 5438
	Analytical Mechanics Associates, Inc.	13, 5144
	Aerospace Corporation	12.2, 5472
	VEGA Deutschland	12, 4873
	University of Strathclyde, Glasgow	12, 5241



	Team name	Value
1.	Politecnico di Torino, Universita' di Roma	311
2.	<i>Advanced Concepts Team (ESA)</i>	308
3.	University of Texas at Austin	267
	DLR	246
	State Key Laboratory & Chinese Academy of Sciences	240
	Analytical Mechanics Associates, Inc.	178
	Tsinghua University	176
	The Aerospace Corp.	163
	University of Colorado, Boulder	154
	U. of Jena, Germany & TU Delft	87
	Beihang University, Beijing	83

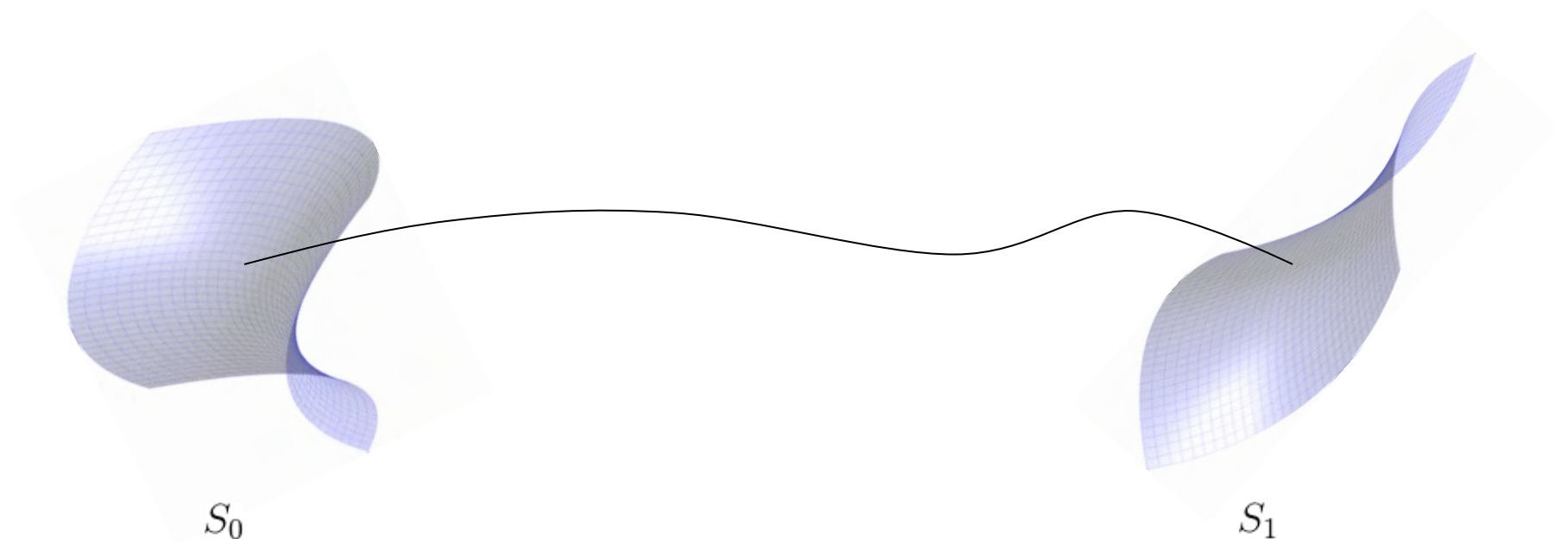




Direct vs. Indirect methods

a. The optimal control problem

minimize: $J(\mathbf{x}(t), \mathbf{u}(t)) = \int_{t_0}^{t_1} F(\mathbf{x}(t), \mathbf{u}(t)) dt$
 subject to: $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))$
 $\mathbf{x}(t_0) \in S_0$
 $\mathbf{x}(t_1) \in S_1$
 $\mathbf{u} \in \mathcal{U}$



$$\mathbf{x} \rightarrow [\mathbf{r}, \mathbf{v}, m]$$

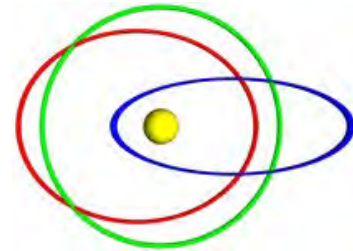
$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \rightarrow \begin{cases} \dot{\mathbf{r}} = \mathbf{v} \\ \dot{\mathbf{v}} = -\frac{\mu}{r^3} \mathbf{r} + c_1 \frac{u}{m} \hat{\mathbf{i}}_u \\ \dot{m} = -c_2 u \end{cases}$$

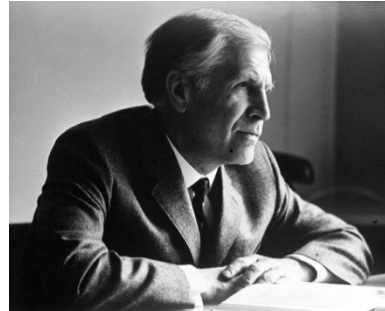
$$\mathbf{x}(t_0) \in S_0 \rightarrow \mathbf{r}(t_0) = \mathbf{r}_0, \quad \mathbf{v}(t_0) = \mathbf{v}_0, \quad m(t_0) = m_0$$

$$\mathbf{x}(t_1) \in S_1 \rightarrow \mathbf{r}(t_1) = \mathbf{r}_1, \quad \mathbf{v}(t_1) = \mathbf{v}_1$$

$$F(\mathbf{x}(t), \mathbf{u}(t)) = -u_1$$

$$\mathbf{u} \in \mathcal{U} \rightarrow u_1 \in [0, 1], \quad |\hat{\mathbf{i}}_u| = 1$$





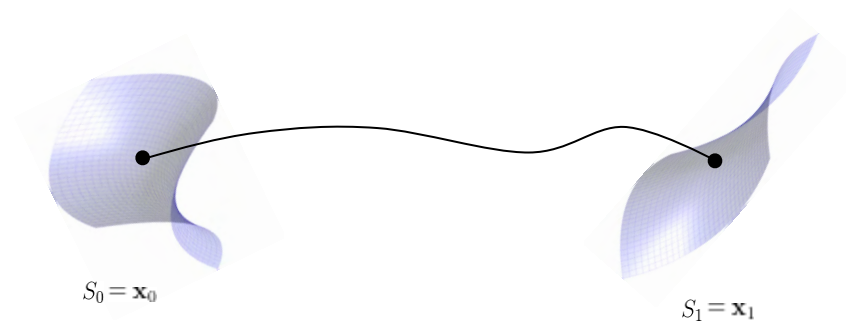
Direct vs. **Indirect** methods

b. Pontryagin Maximum Principle

$$\mathcal{H}(\boldsymbol{\lambda}(t), \mathbf{x}(t), \mathbf{u}(t)) = \boldsymbol{\lambda}(t) \cdot \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) - F(\mathbf{x}(t), \mathbf{u}(t))$$

$$\dot{\mathbf{x}} = \frac{d\mathcal{H}}{d\boldsymbol{\lambda}}$$

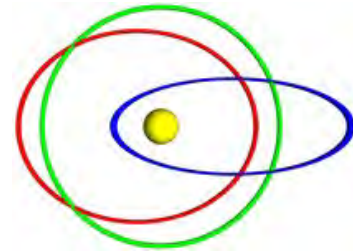
$$\dot{\boldsymbol{\lambda}} = -\frac{d\mathcal{H}}{d\mathbf{x}}$$



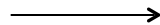
$$(i) \quad \mathcal{H}(\boldsymbol{\lambda}(t), \mathbf{x}(t), \mathbf{u}^*(t)) \geq \mathcal{H}(\boldsymbol{\lambda}(t), \mathbf{x}(t), \mathbf{u}(t)), \quad \forall \mathbf{u}(t) \in \mathcal{U}$$

$$(ii) \quad \mathcal{H}(\boldsymbol{\lambda}(t_1), \mathbf{x}(t_1), \mathbf{u}^*(t_1)) = 0$$

$$\mathcal{H}(\boldsymbol{\lambda}(t), \mathbf{x}(t), \mathbf{u}(t)) = \boldsymbol{\lambda}_r(t) \cdot \mathbf{v}(t) + \boldsymbol{\lambda}_v(t) \cdot \left(-\frac{\mu}{r^3} \mathbf{r} + c_1 \frac{u}{m} \hat{\mathbf{i}}_u \right) - \lambda_m c_2 u - u$$

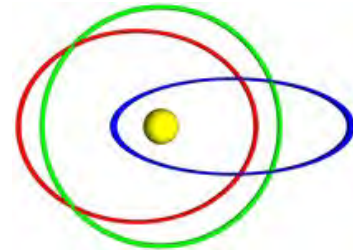


$$\begin{aligned} \dot{\mathbf{x}} &= \frac{d\mathcal{H}}{d\boldsymbol{\lambda}} \\ \dot{\boldsymbol{\lambda}} &= -\frac{d\mathcal{H}}{d\mathbf{x}} \end{aligned}$$



$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= -\frac{\mu}{r^3} \mathbf{r} + c_1 \frac{u}{m} \hat{\mathbf{i}}_u \\ \dot{m} &= -c_2 u \\ \dot{\boldsymbol{\lambda}}_r &= \frac{\mu}{r^3} \boldsymbol{\lambda}_v - \frac{3\mu \mathbf{r} \cdot \boldsymbol{\lambda}_v}{r^5} \mathbf{r} \\ \dot{\boldsymbol{\lambda}}_v &= -\boldsymbol{\lambda}_r \\ \dot{\lambda}_m &= -c_1 \frac{u}{m^2} \boldsymbol{\lambda}_v \cdot \hat{\mathbf{i}}_u \end{aligned}$$

$$\mathcal{H}(\boldsymbol{\lambda}(t), \mathbf{x}(t), \mathbf{u}(t)) = \boldsymbol{\lambda}_r(t) \cdot \mathbf{v}(t) + \boldsymbol{\lambda}_v(t) \cdot \left(-\frac{\mu}{r^3} \mathbf{r} + c_1 \frac{u}{m} \hat{\mathbf{i}}_u \right) - \lambda_m c_2 u - u$$

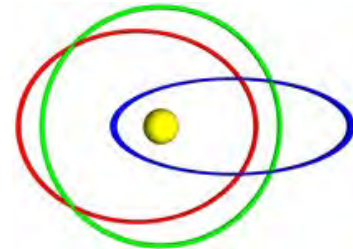


Let us apply Pontryagin maximum principle:

$$\hat{\mathbf{i}}_u = \frac{\boldsymbol{\lambda}_v}{\lambda_v} \quad \begin{cases} u = 0 & \rho < 0 \\ u = 1 & \rho > 0 \\ u \in [0, 1] & \rho = 0 \end{cases}$$

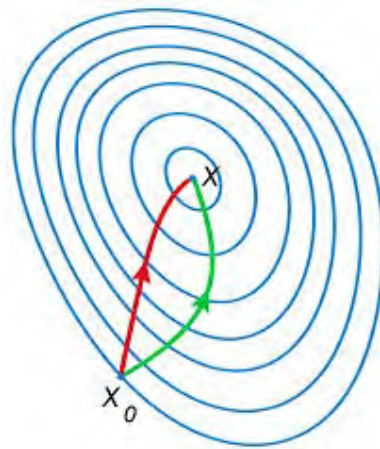
$$\rho = \frac{c_1}{m} \lambda_v - c_2 \lambda_m - 1 \quad \text{Switching function}$$

$$\mathcal{H}(\boldsymbol{\lambda}(t), \mathbf{x}(t), \mathbf{u}(t)) = \boldsymbol{\lambda}_r(t) \cdot \mathbf{v}(t) + \boldsymbol{\lambda}_v(t) \cdot \left(-\frac{\mu}{r^3} \mathbf{r} + c_1 \frac{u}{m} \hat{\mathbf{i}}_u \right) - \lambda_m c_2 u - u$$



$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \frac{\mu}{r^3} \mathbf{r} + c_1 \frac{u}{m} \frac{\boldsymbol{\lambda}_v}{\lambda_v} \\ \dot{m} &= -c_2 u \\ \dot{\boldsymbol{\lambda}}_r &= \frac{\mu}{r^3} \boldsymbol{\lambda}_v - 3\mu \frac{\mathbf{r} \cdot \boldsymbol{\lambda}_v}{r^5} \mathbf{r} \\ \dot{\boldsymbol{\lambda}}_v &= -\boldsymbol{\lambda}_r \\ \dot{\lambda}_m &= -c_1 \frac{u}{m^2} \lambda_v \end{aligned} \quad \left\{ \begin{array}{ll} u = 0 & \rho < 0 \\ u = 1 & \rho > 0 \\ u \in [0, 1] & \rho = 0 \end{array} \right. \quad \rho = \frac{c_1}{m} \lambda_v - c_2 \lambda_m - 1$$

The OCP becomes a **Two Points Boundary Value Problem** TPBVP: find the costate initial values so that the final states are achieved at the final time (plus final condition on the hamiltonian if time is free, plus transversality conditions)



Direct vs. Indirect methods

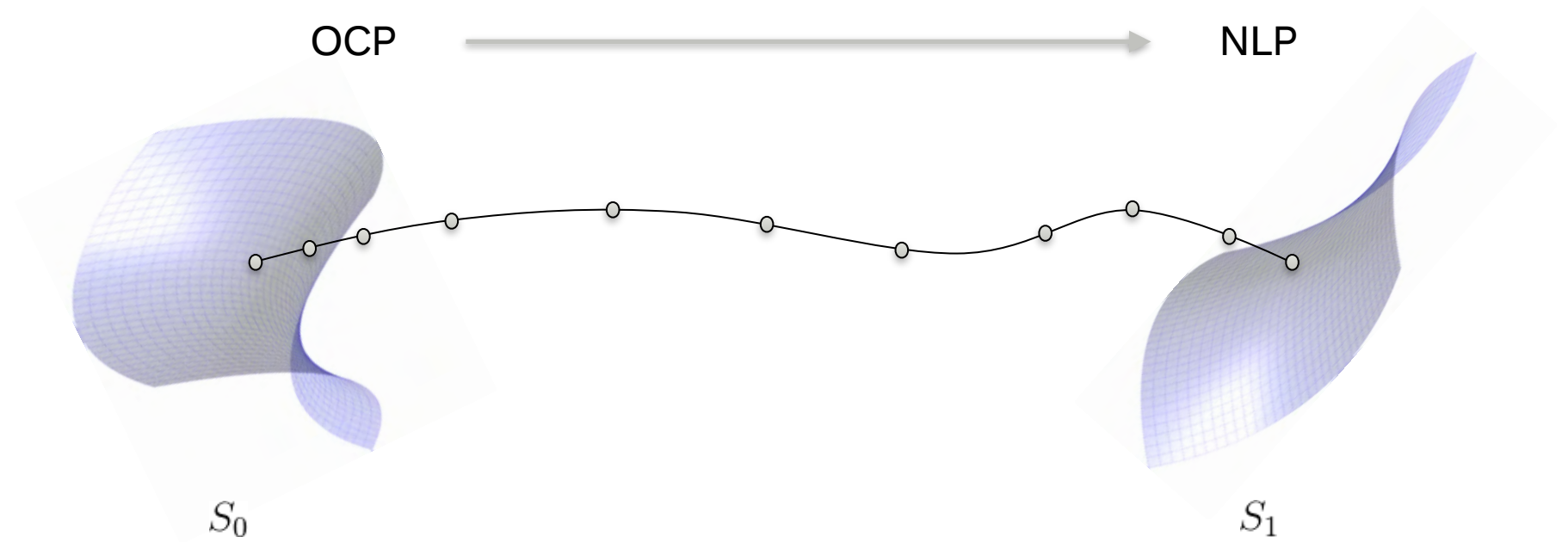
b. Transcription Methods

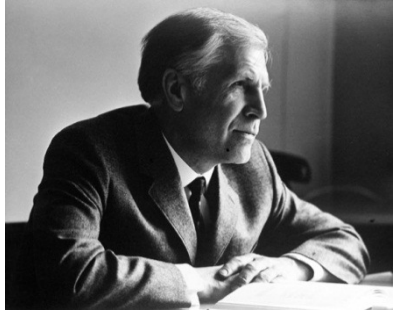
minimize: $J(\mathbf{x}(t), \mathbf{u}(t)) = \int_{t_0}^{t_1} F(\mathbf{x}(t), \mathbf{u}(t)) dt$
 subject to: $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))$
 $\mathbf{x}(t_0) \in S_0$
 $\mathbf{x}(t_1) \in S_1$
 $\mathbf{u} \in \mathcal{U}$

minimize: $J(\mathbf{y})$
 subject to: $\mathbf{g}(\mathbf{y}) \leq 0$

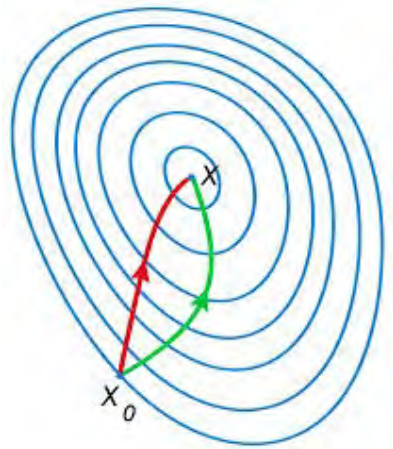
OCP

NLP



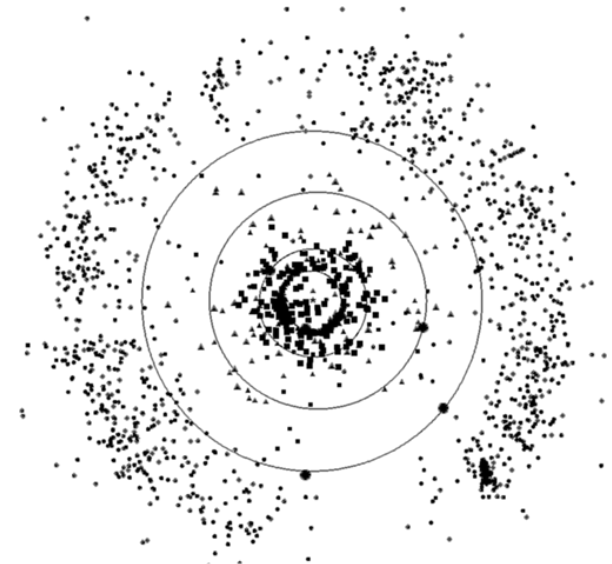
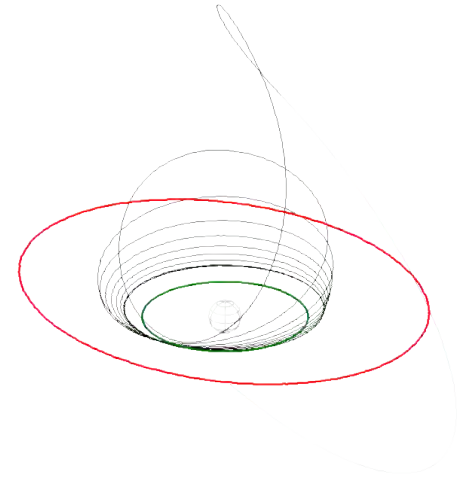
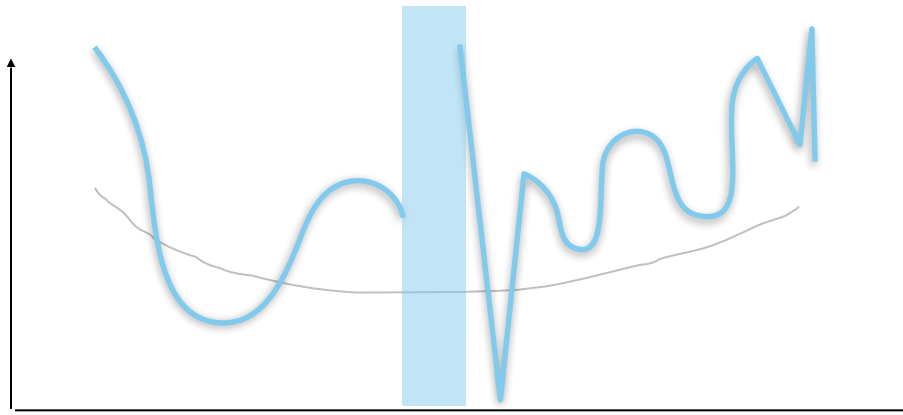


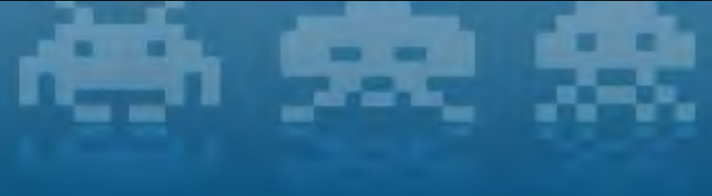
Indirect methods: OCP -> TPBVP



Direct methods: OCP -> NLP

find: $\mathbf{x} \in \mathbb{R}^n \times \mathbb{N}^m$
 to minimize: $f(\mathbf{x}) : \mathbb{R}^n \times \mathbb{N}^m \rightarrow \mathbb{R}^p$
 subject to: $\mathbf{g}(\mathbf{x}) \leq 0$

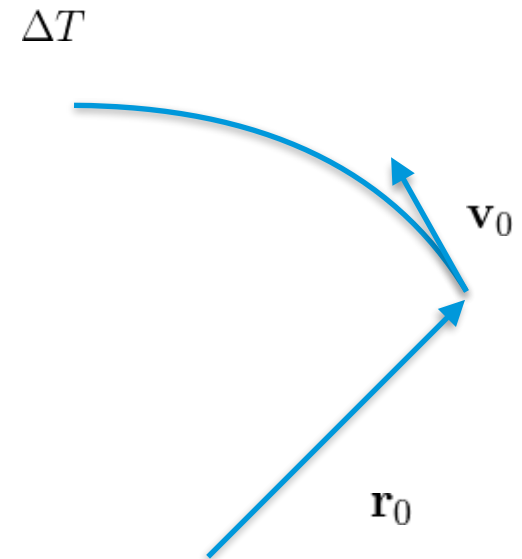




Building blocks

ii. Lagrange propagation

1. Predicting the time evolution of an orbit from starting conditions
2. It is an initial value problem (Cauchy)
3. Its solution can be efficiently obtained in terms of the Lagrange coefficients F,G
4. Kepler's equation needs to be solved to invert the eccentric anomaly - time relation.



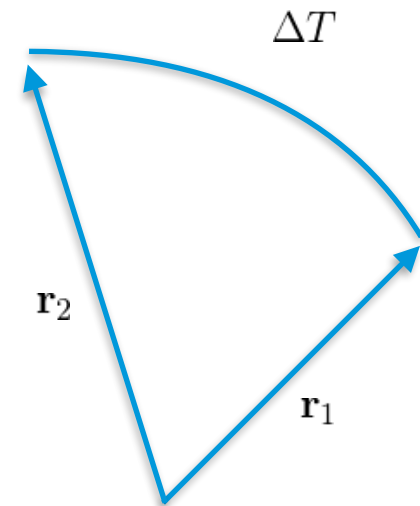
$$\begin{cases} \dot{\mathbf{r}} = \mathbf{v} \\ \dot{\mathbf{v}} = -\frac{\mu}{r^3}\mathbf{r} \\ \mathbf{r}(0) = \mathbf{r}_0 \\ \mathbf{v}(0) = \mathbf{v}_0 \end{cases}$$



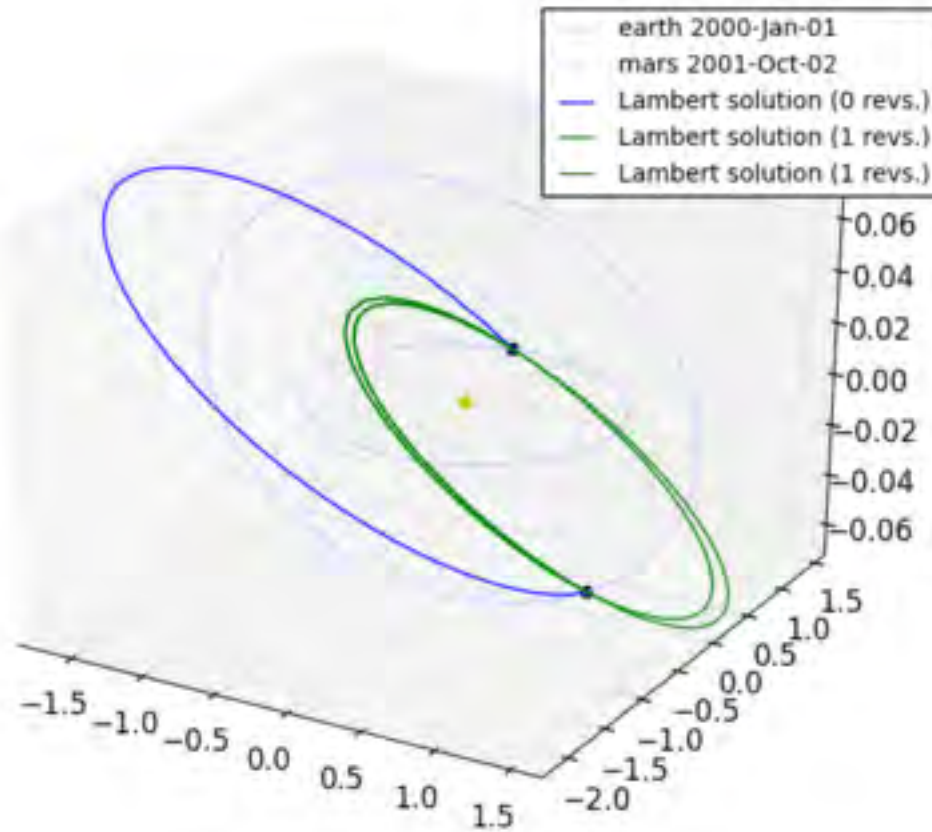
Building blocks

i. Lambert's Problem

1. Going from one point to another in a fixed time.
2. It is, again, a TPBVP.
3. Its modern solution relies on results from Lambert, Gauss, Lagrange and in more modern times Battin, Lancaster and Blanchard
4. It turns out that all Lambert problems have 1 solution and, according to the transfer time, may also have $2*N$ multiple revolution solutions.



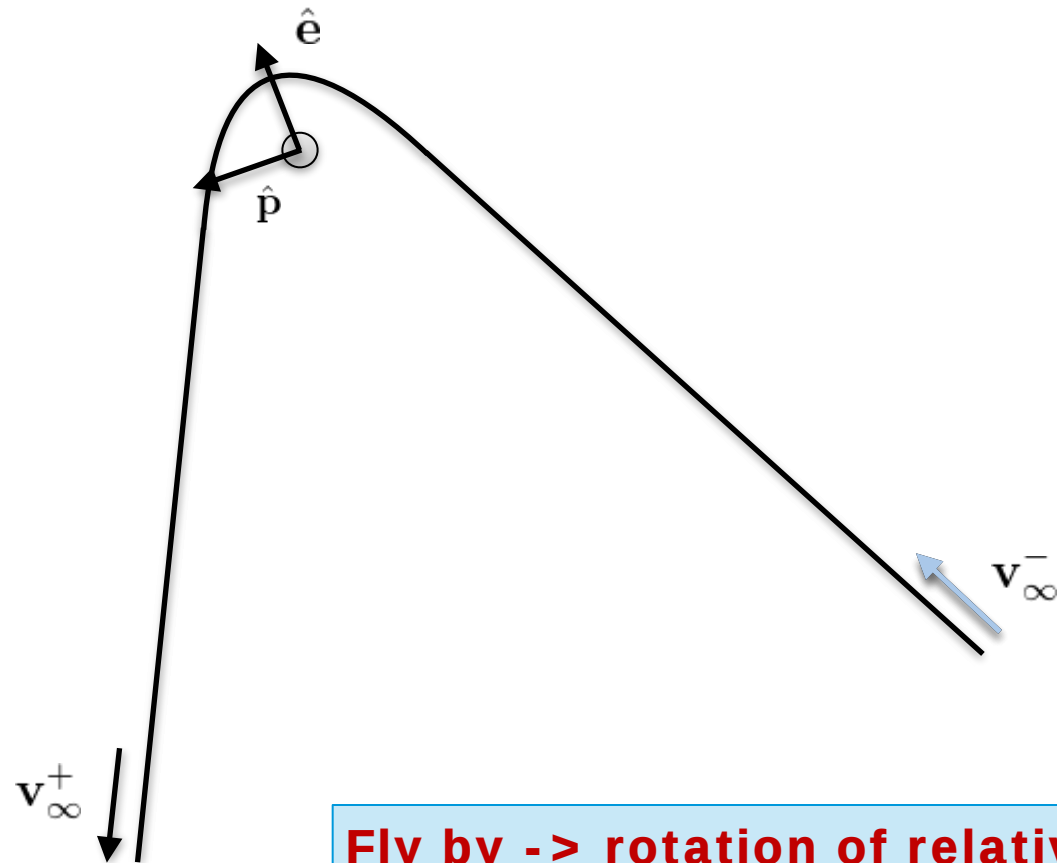
$$\begin{cases} \dot{\mathbf{r}} = \mathbf{v} \\ \dot{\mathbf{v}} = -\frac{\mu}{r^3}\mathbf{r} \\ \mathbf{r}(0) = \mathbf{r}_1 \\ \mathbf{r}(T) = \mathbf{r}_2 \end{cases}$$



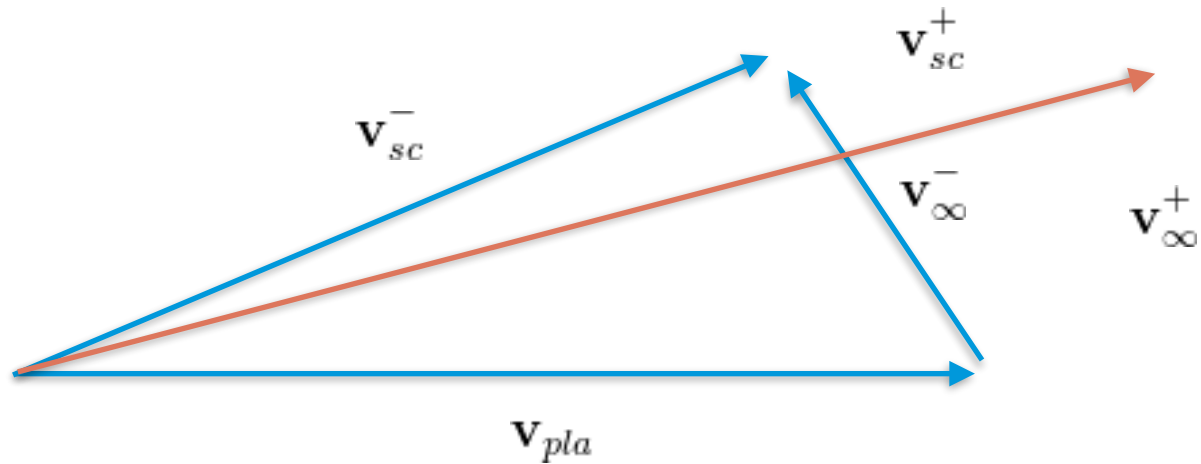


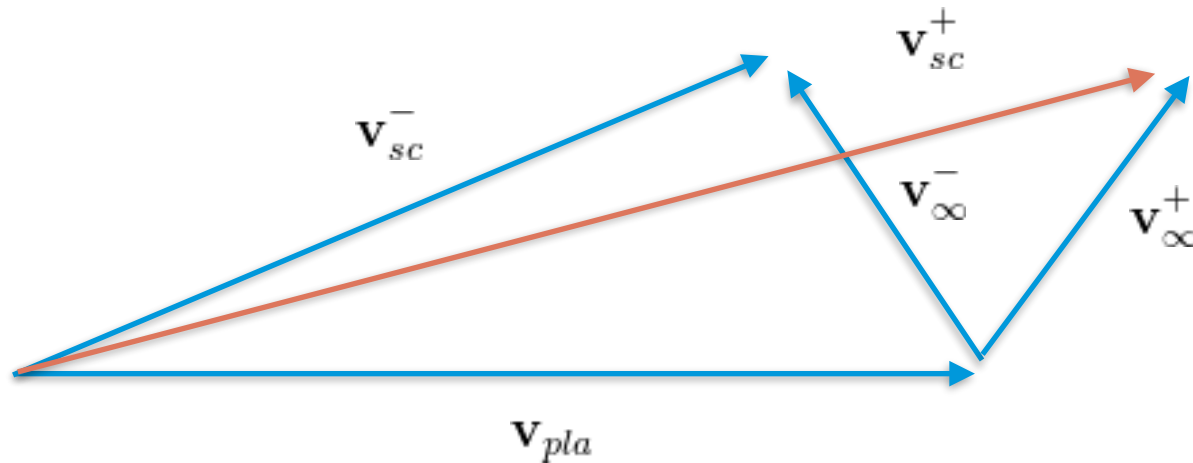
Building blocks

iii. Mivovitch Fly-bys



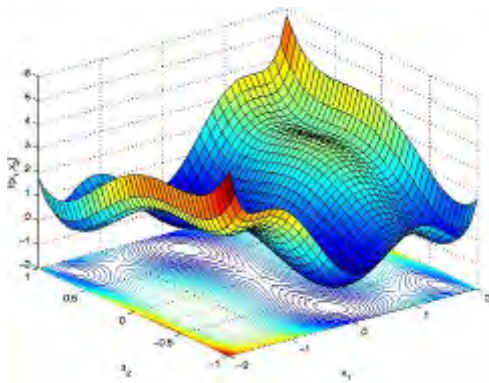
Fly by -> rotation of relative velocity





Spacecraft velocity has changed in the absolute frame

Watch Mivovitch [Video](#)



A deeper look into direct methods

i. Chemical propulsion

Given a planetary sequence of N planets find:

$$x = [t_{dep}, T_1, \dots, T_N, T_{arr}]$$

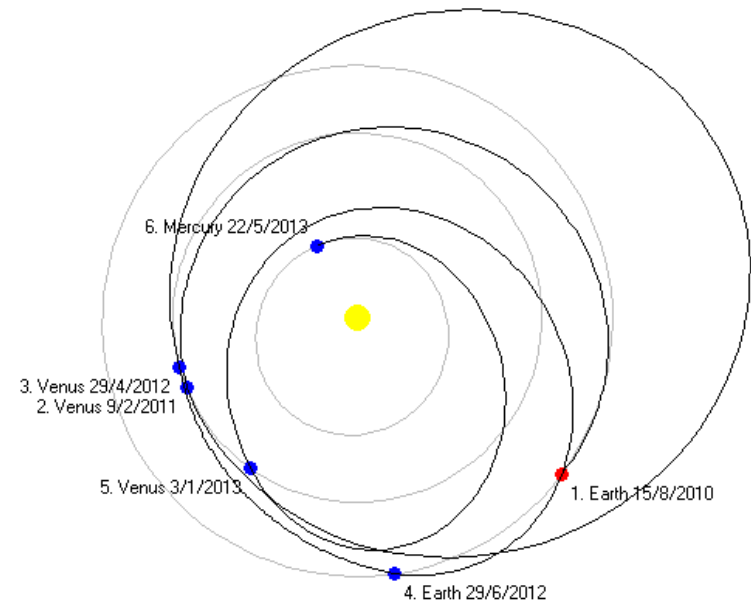
To minimise:

$$J = \Delta V_1 + \Delta V_2 + \dots + \Delta V_{arr} (+\Delta V_{dep})$$

Subject to:

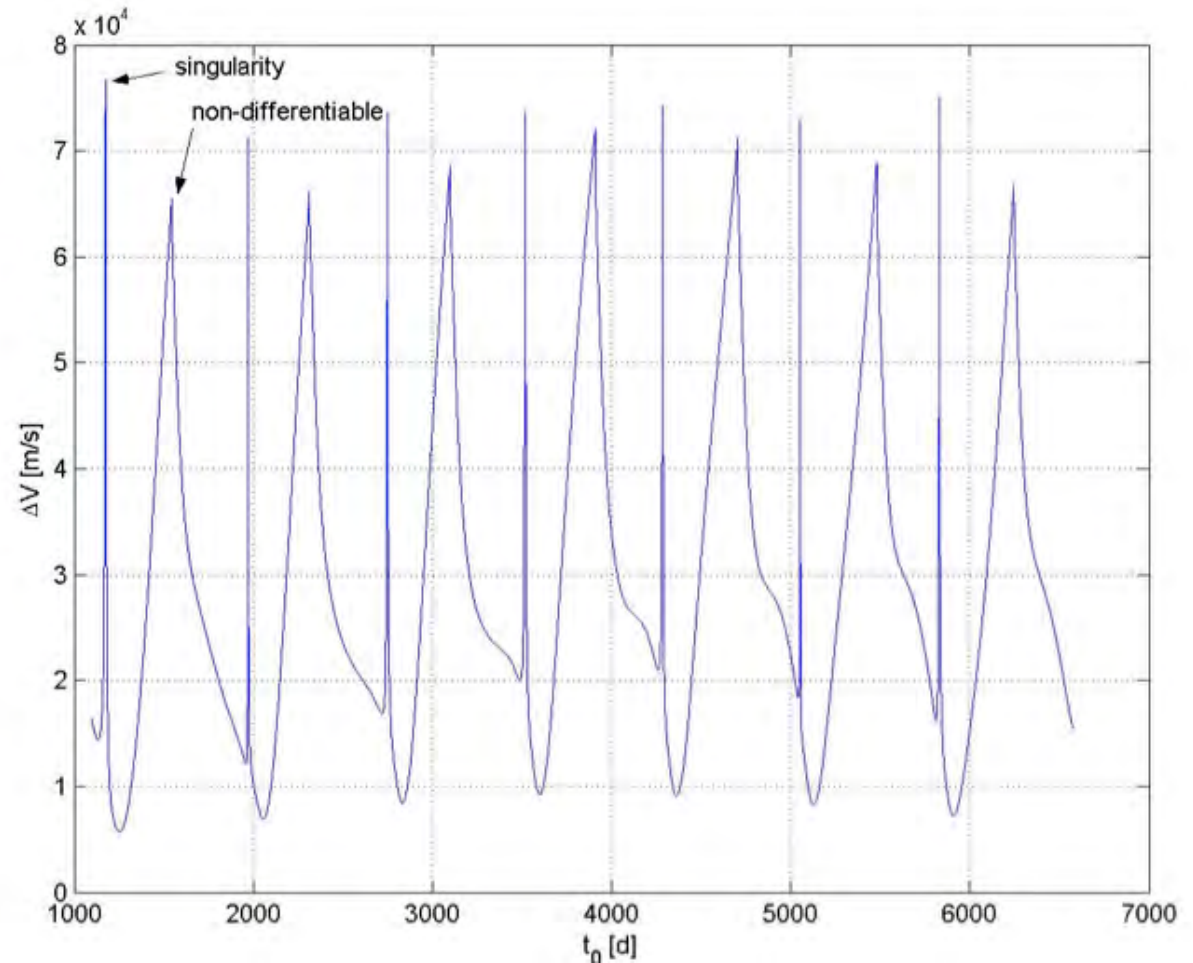
$$x \in [x, \bar{x}] \quad \text{Launch window constraint}$$

$$(\Delta V_{dep}^2 < C3_{launch}) \quad \text{Launcher constraint}$$

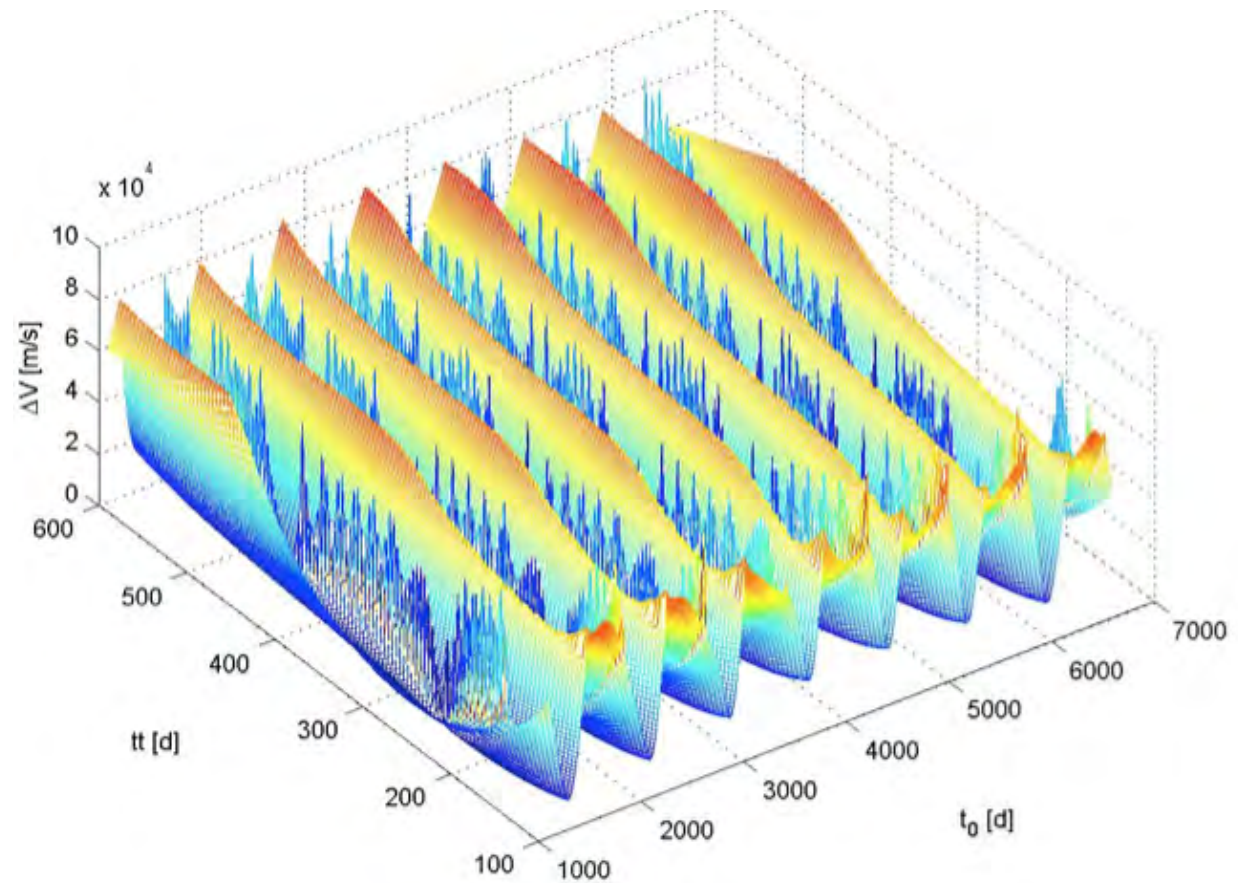


Planetary sequence: EVVEVMe

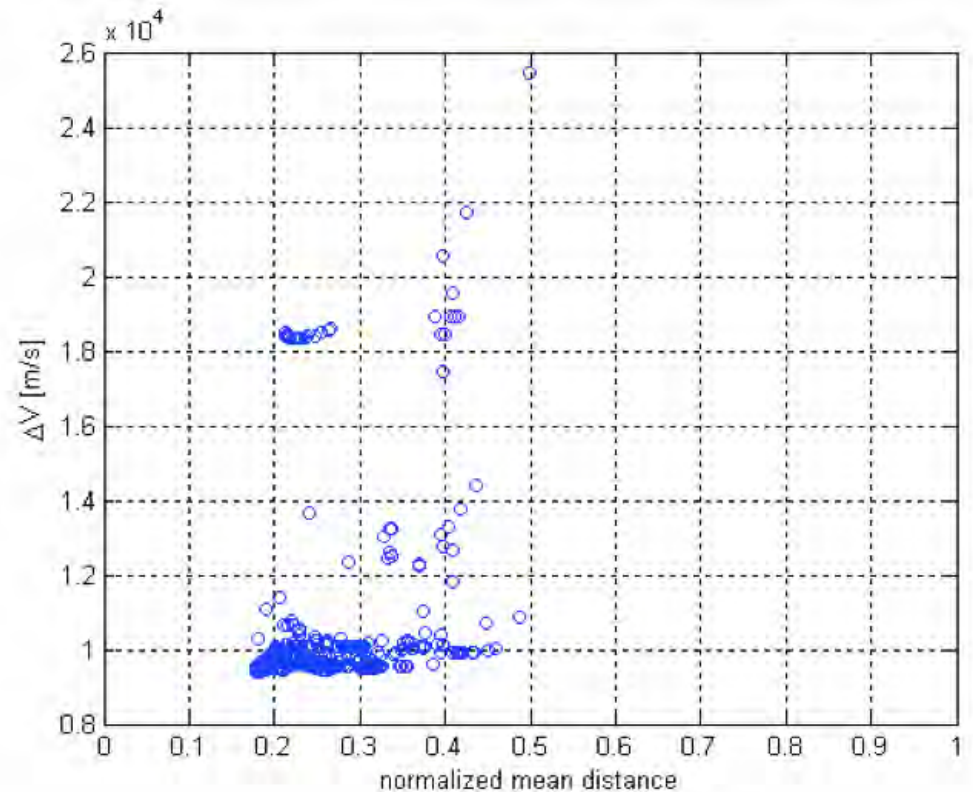
1. Earth-Mars transfer
2. Chemical propulsion
3. 200 days of transfer
4. MJD2000 used



1. Earth-Mars transfer
2. Chemical propulsion
3. Days and MJD2000 used

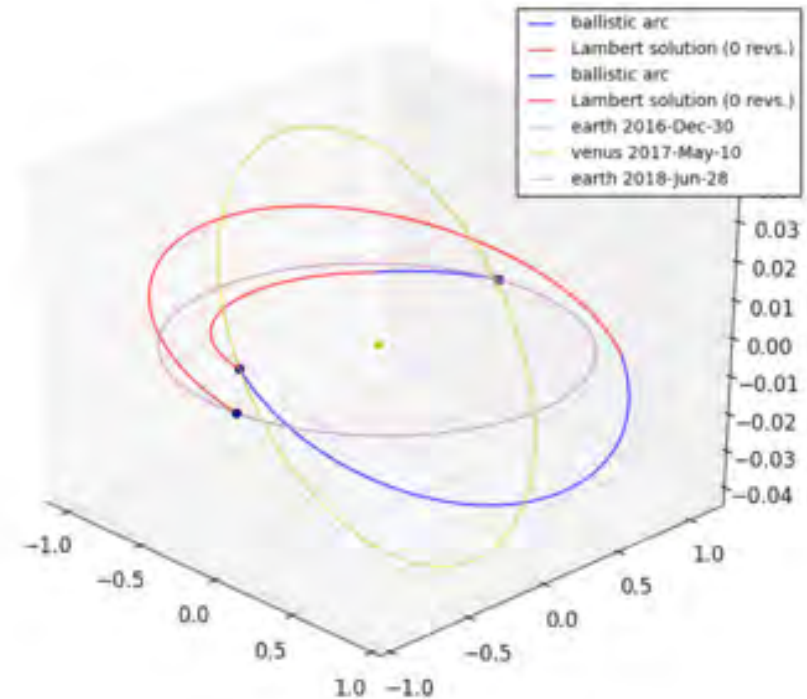


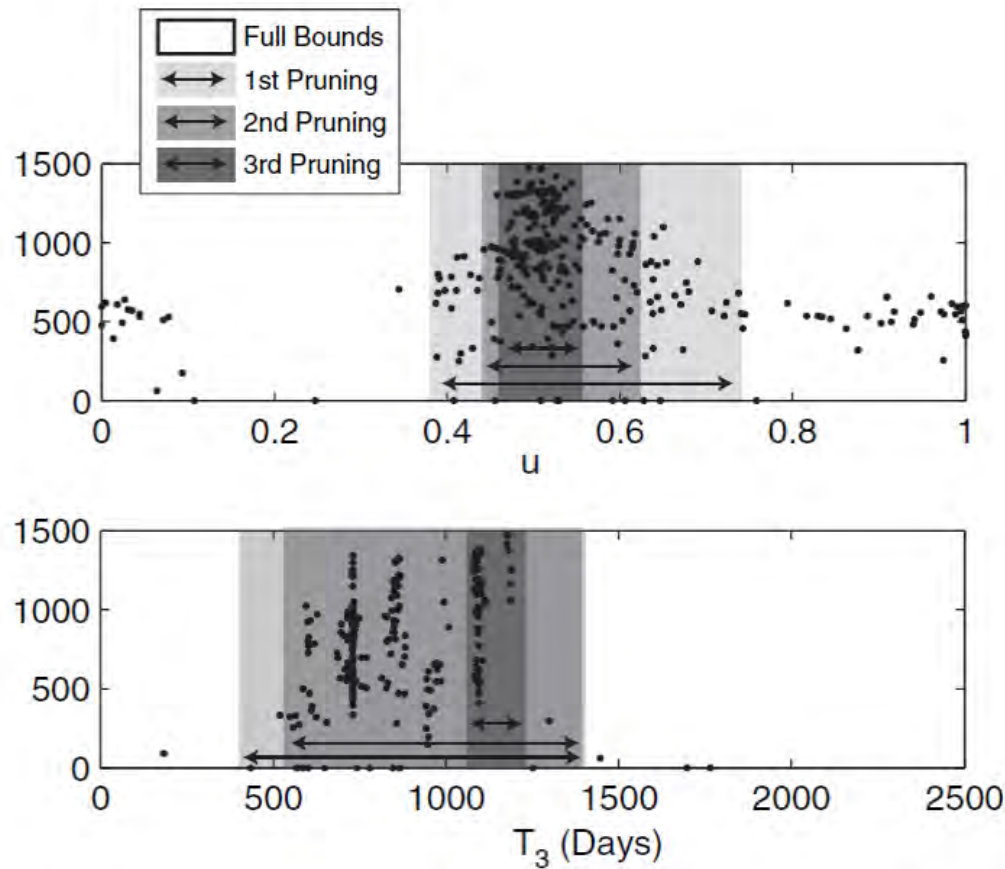
1. Earth-Jupiter-Saturn transfer
2. Local optima cluster together
3. Better local optima are close to the global one
4. Clustered local optima have similar objective values



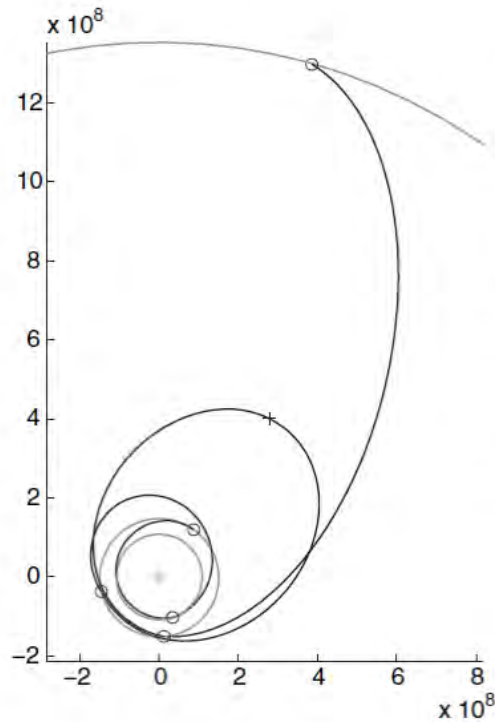
$$\mathbf{x} = [t_0, V_\infty, u, v, \eta, T_0] + \dots + [r_p, \beta, \eta_i, T_i]$$

- Features of the MGA-1DSM model:
 - DSM value can be zero
 - Multi-revs are included
 - Resonant returns and backflips included
- Multiple objectives can be handled (http://sophia.estec.esa.int/gtocwiki/index.php/Final_Solution)





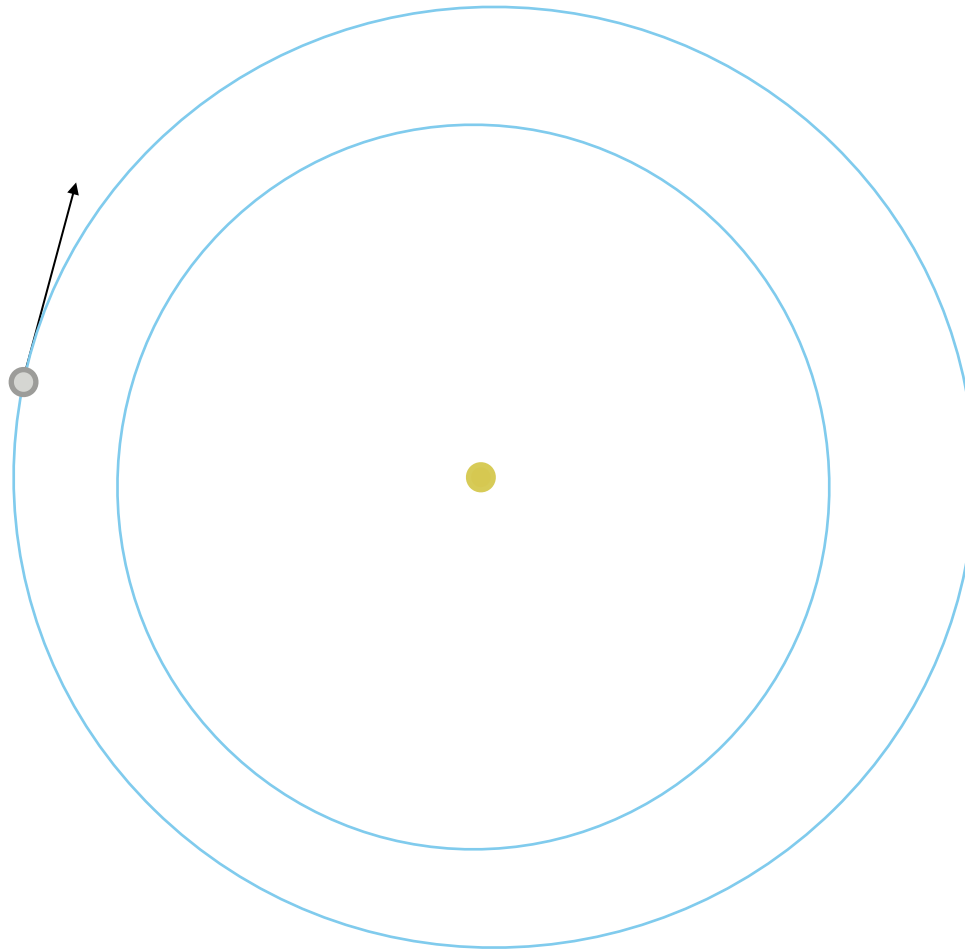
Developed as part of the
work for the joint JPL/ESA
Outer Planets Mission
Analysis Working Group
(2007-2008)



Spacecraft	
Departure mass	2085.44 kg
Arrival mass	1476.03 kg
I_{sp}	312 s
Departure	
Epoch	15/11/2021
V_{∞}	3.34 km/s
Declination	3.1 deg
Cruise	
Venus fly-by	30/04/2022
Earth fly-by	04/04/2023
DSM Epoch	13/01/2025
DSM ΔV	167 m/s
Earth fly-by	25/06/2026
Arrival	
Epoch	03/07/2031
V_{∞}	0.676 km/s
Total flight time	9.63 years

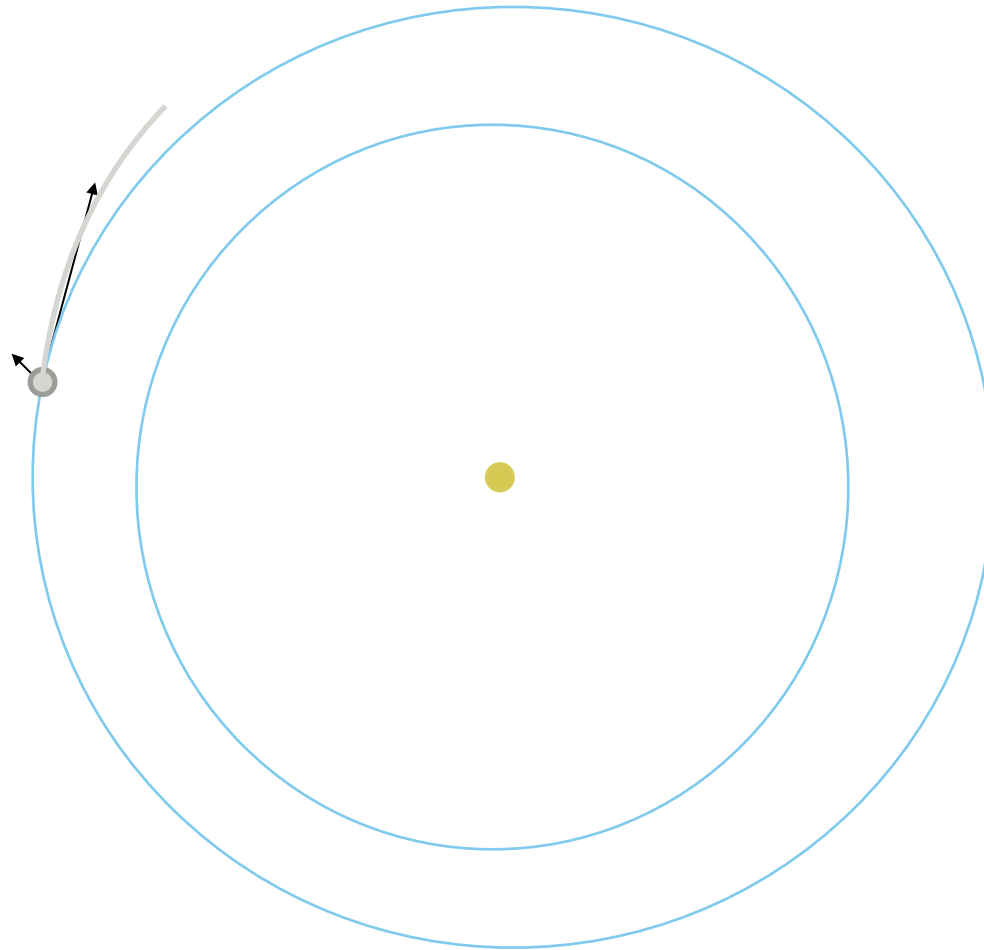


t_0

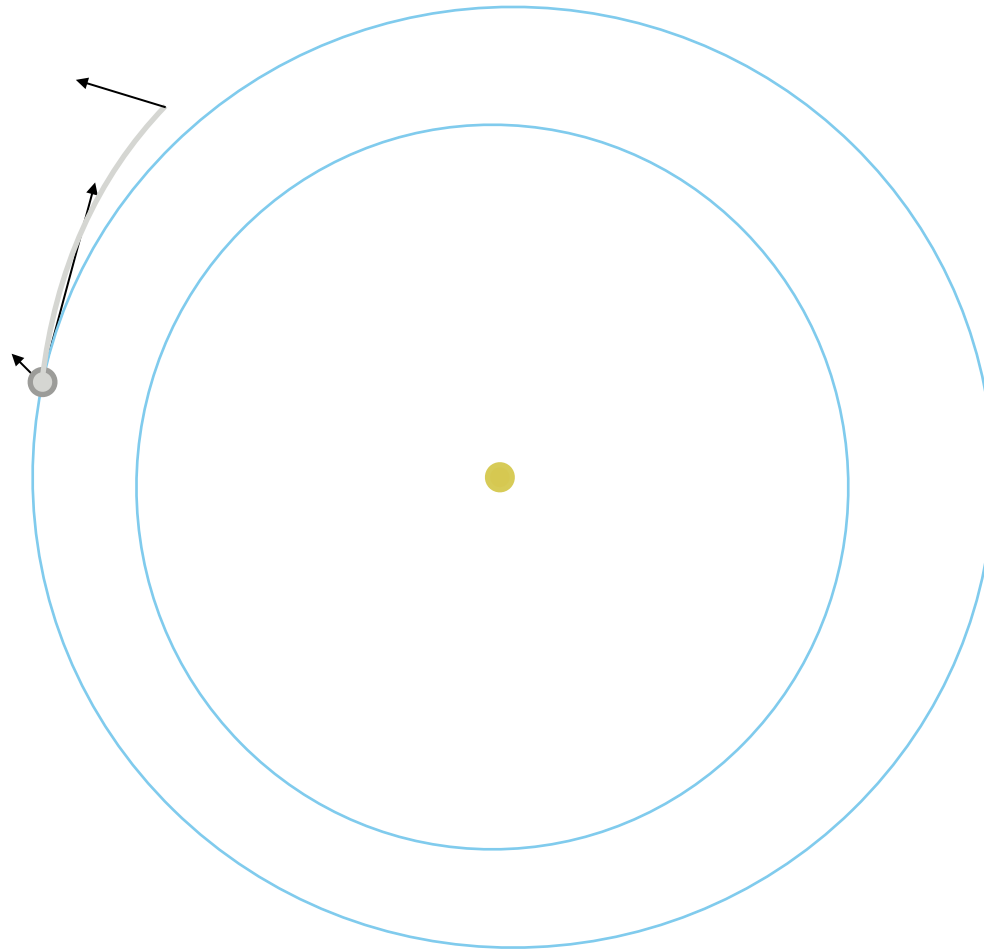




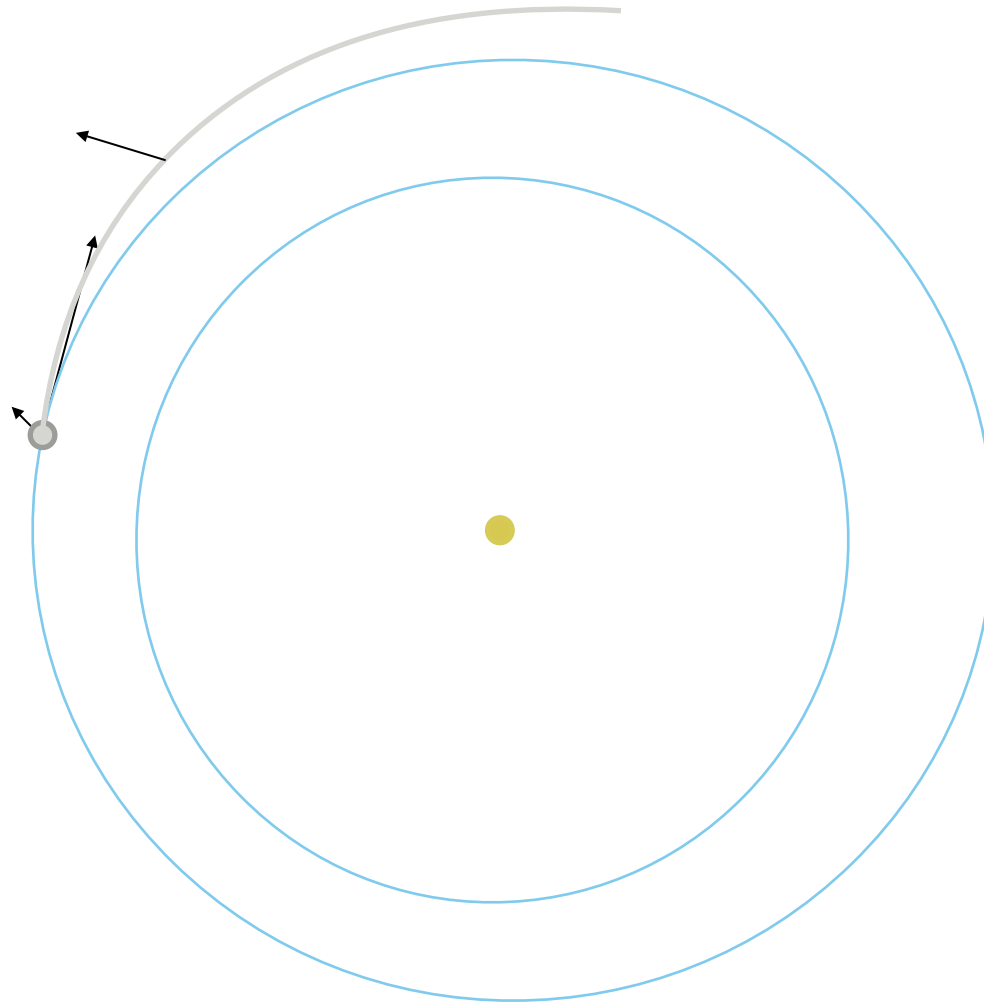
$t_0, \mathbf{v}_\infty$

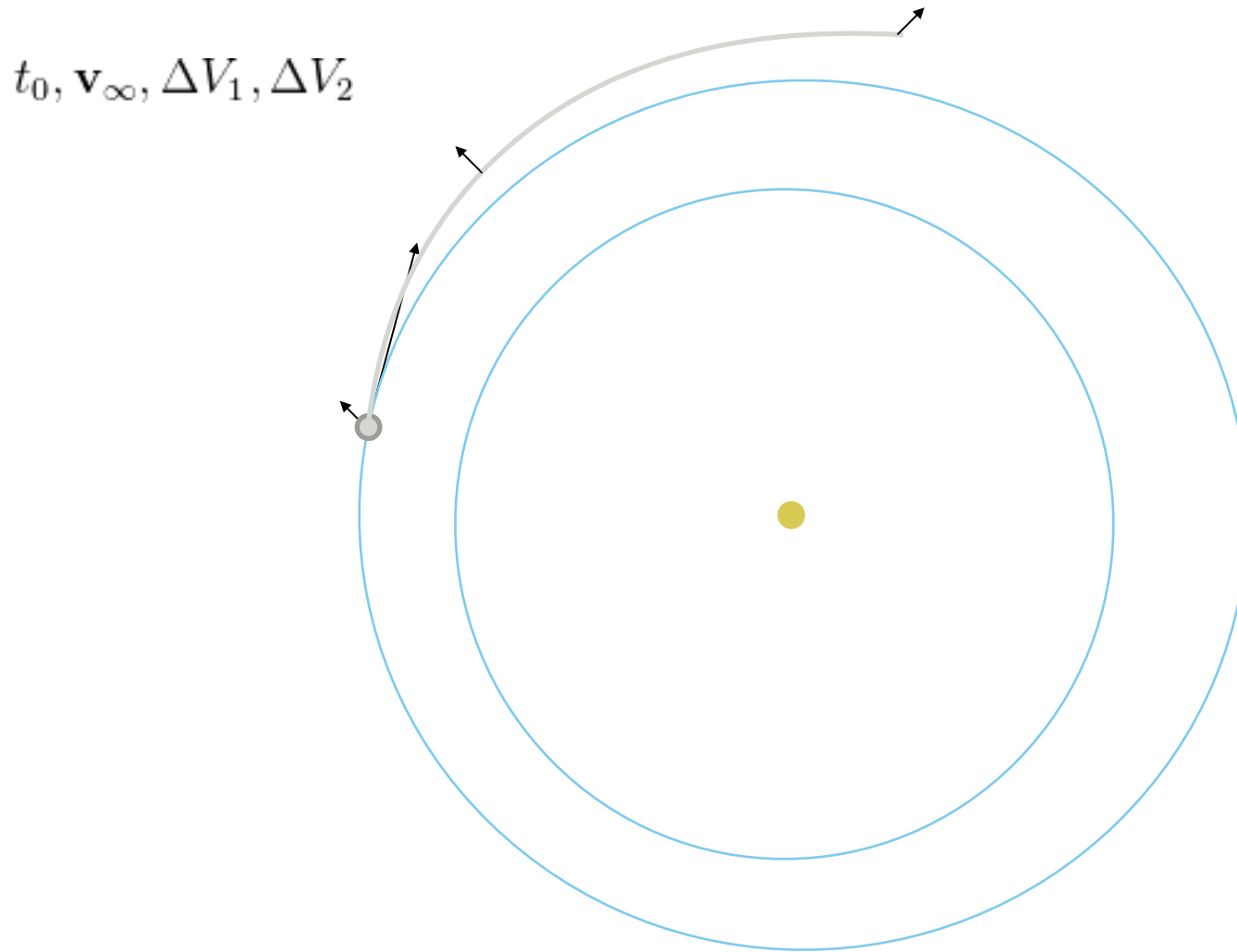


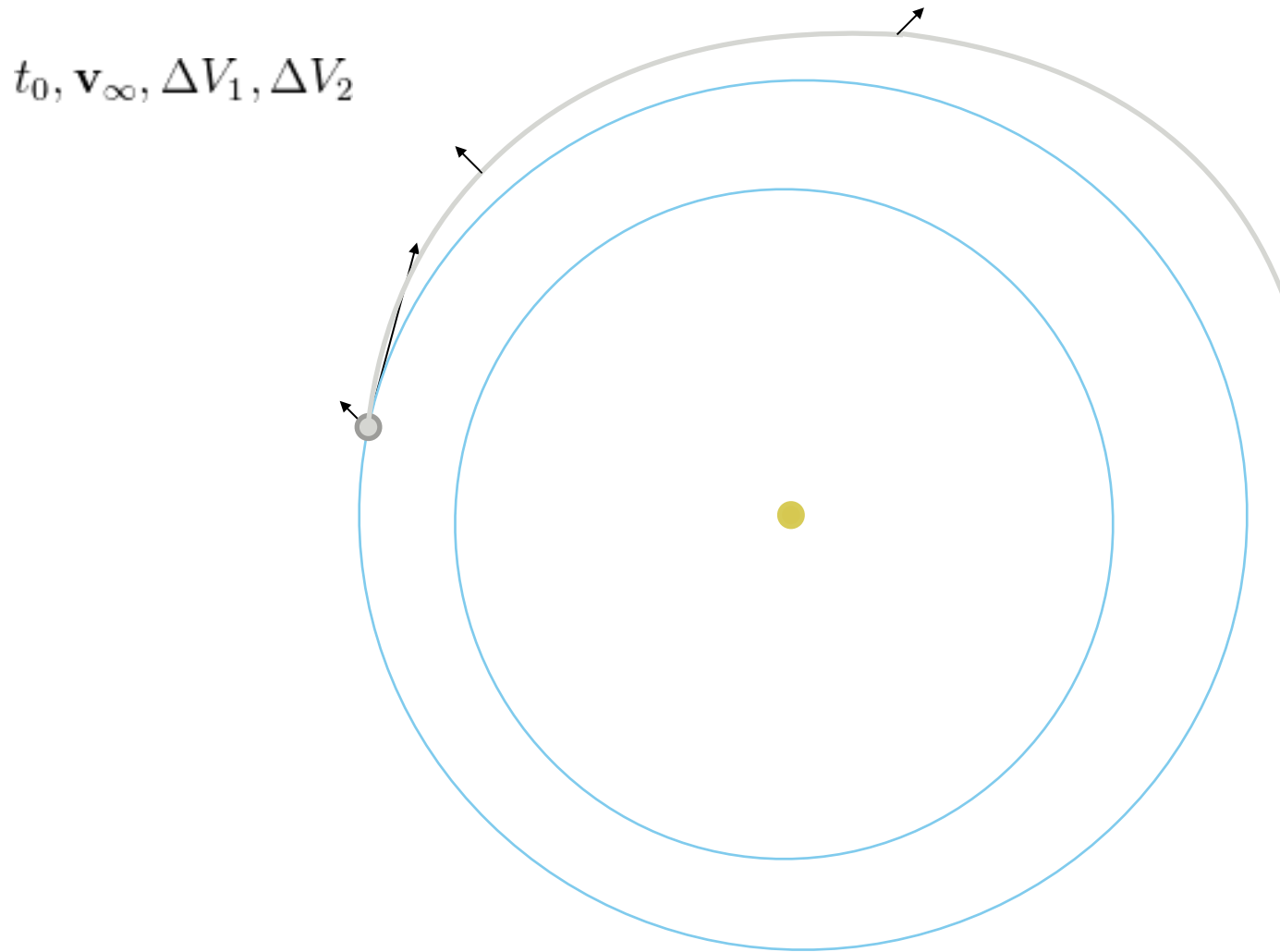
$t_0, \mathbf{v}_\infty, \Delta V_1$



$t_0, \mathbf{v}_\infty, \Delta V_1$





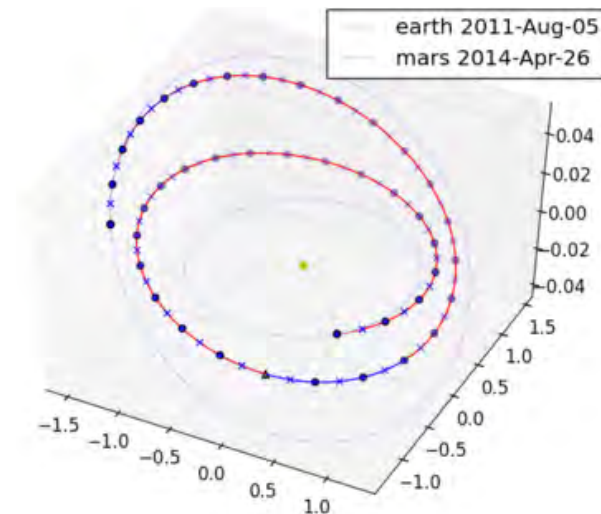
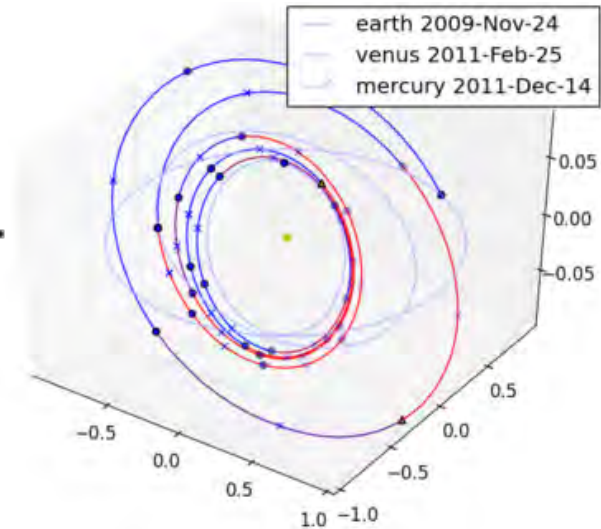


$$\begin{aligned}
 x = & [t_0] \\
 & + [T_1, m_{f1}, V_{xi1}, V_{yi1}, V_{zi1}, V_{xf1}, V_{yf1}, V_{zf1}] \\
 & + [T_2, m_{f2}, V_{xi2}, V_{yi2}, V_{zi2}, V_{xf2}, V_{yf2}, V_{zf2}] + \dots \\
 & + [u_x^1, u_y^1, u_z^1] + [u_x^2, u_y^2, u_z^2] + \dots
 \end{aligned}$$

constraints:

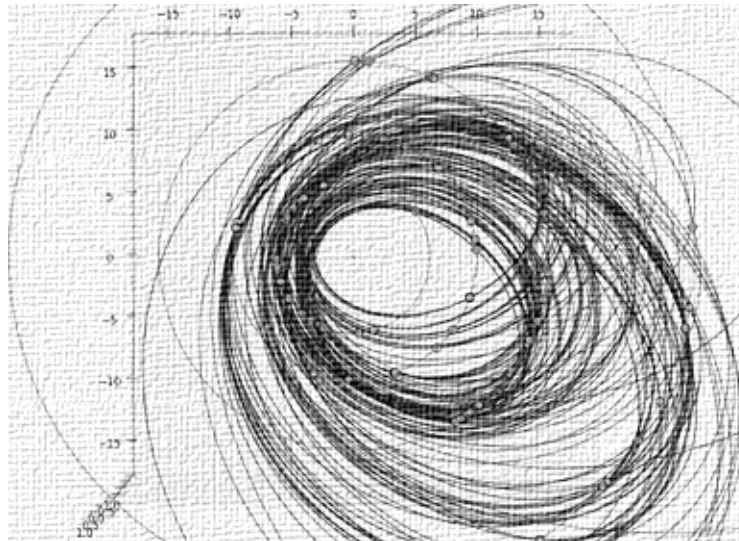
$$\begin{aligned}
 & \text{mismatch} \\
 & |u^i| \\
 & |V_{f_i}| = |V_{f_{i+1}}| \\
 & |V_{f_i} \cdot V_{f_{i+1}}| \geq \alpha
 \end{aligned}$$

- Features of the MGA-LT model:
 - Easy switch between low and high fidelity
 - Large convergence radius





On line resources



GTOP - A Trajectory Problem Database



Crowdsourcing Experiments (html5)

The Space Game
Jupiter Hopper

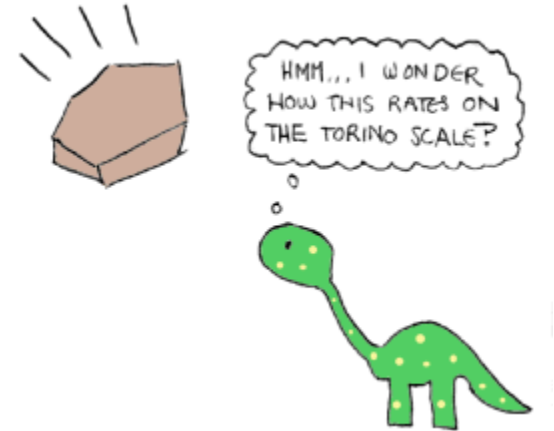


SOCIS
PyKEP
PyGMO



Asteroid deflection: What the dinosaurs did not know

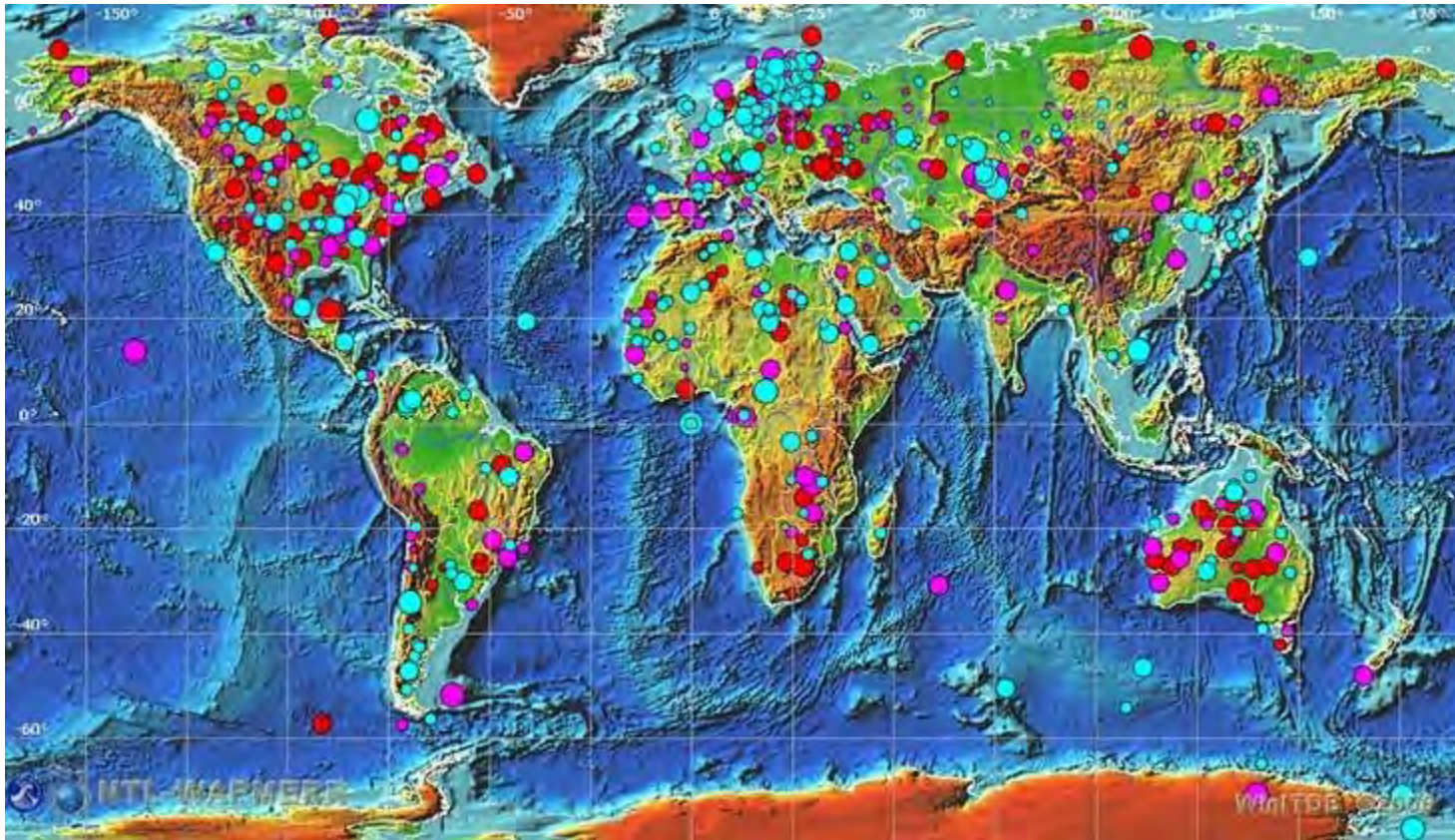
- Impacts happen!!!
- From mass extinctions to regional to local damage
- We know of ~500,000 asteroids in the solar system
- ~800 are potentially hazardous
- What the dinosaurs did not know ([video](#))



Tunguska 1908 (icy comet? Small asteroid?)



Winslow, Arizona (~49.000 years ago)



Map of confirmed (red), perspective for verification (magenta) and proposed for further study (blue) impact structures on the Earth. Size of circles is proportional to the crater diameter.

Source: Institute of Computational Mathematics and Mathematical Geophysics. Novosibirsk, RUSSIA

- 2003: 6 industrial studies on the topic of NEOs characterization and hazard mitigation
- 2004: DQ is recommended as highest priority for near-term implementation of NEO missions (NEOMAP): “Don Quijote has the potential to teach us a great deal, not only about the internal structure of a NEO, but also about how to mechanically interact with it”
- First time that the deflection of a celestial body may be proved and studied

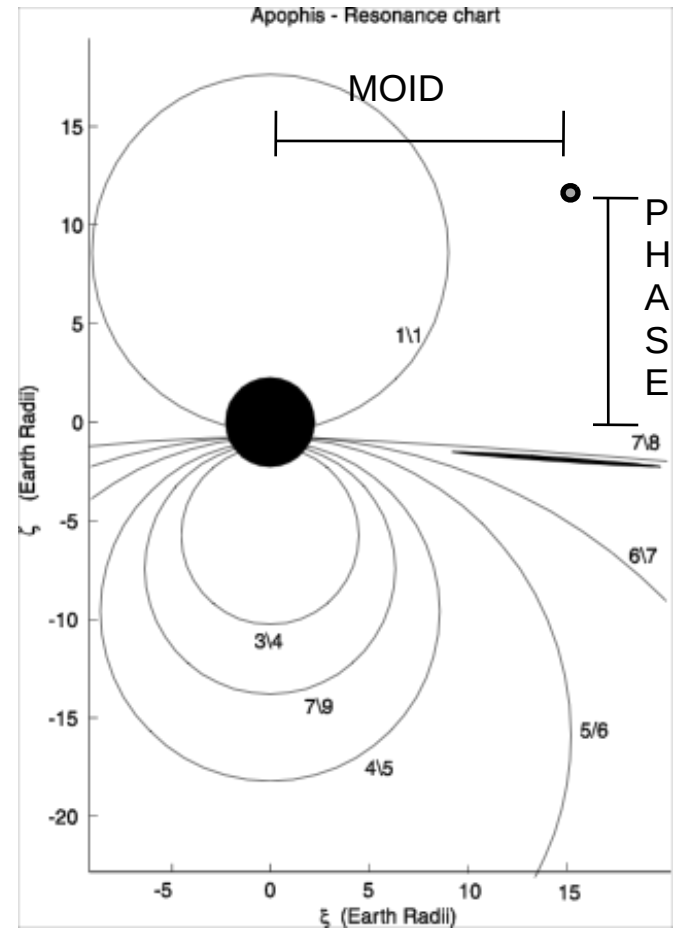
The orbital mechanics of moving asteroids needed to be studied in more details



Sancho Observing the impact

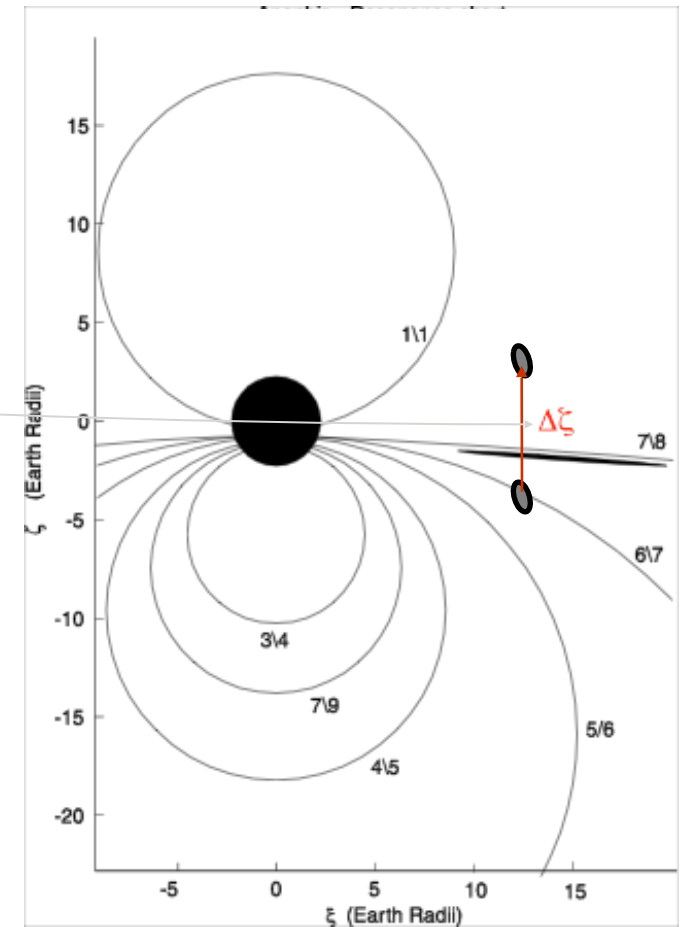


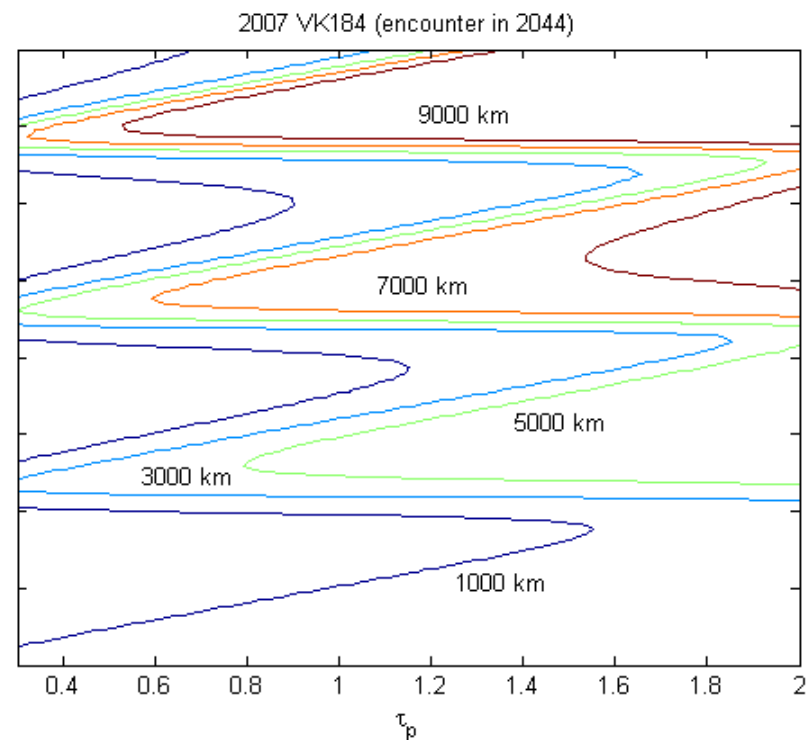
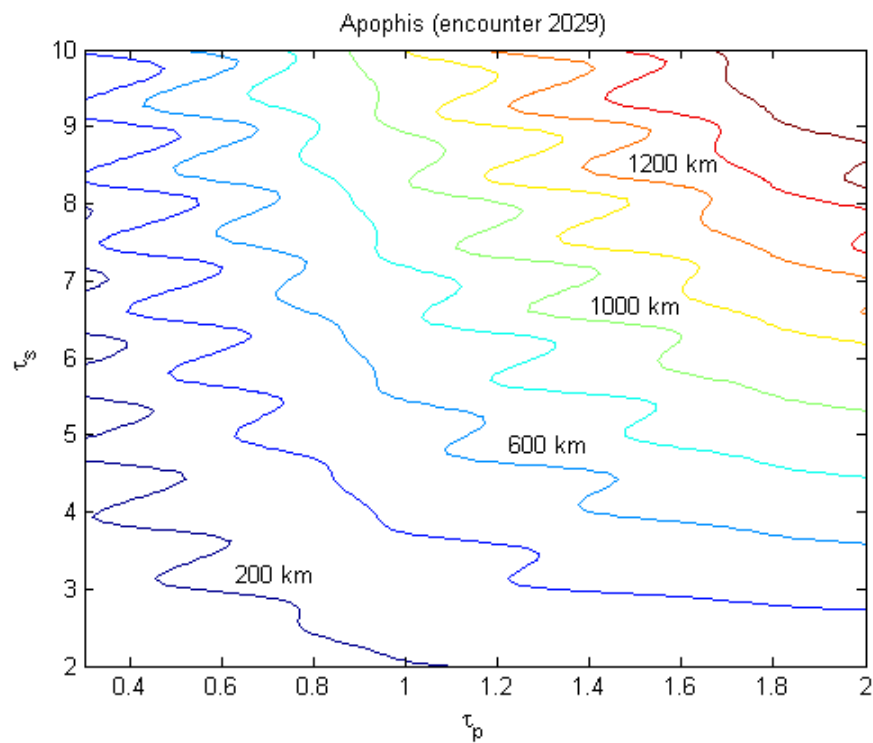
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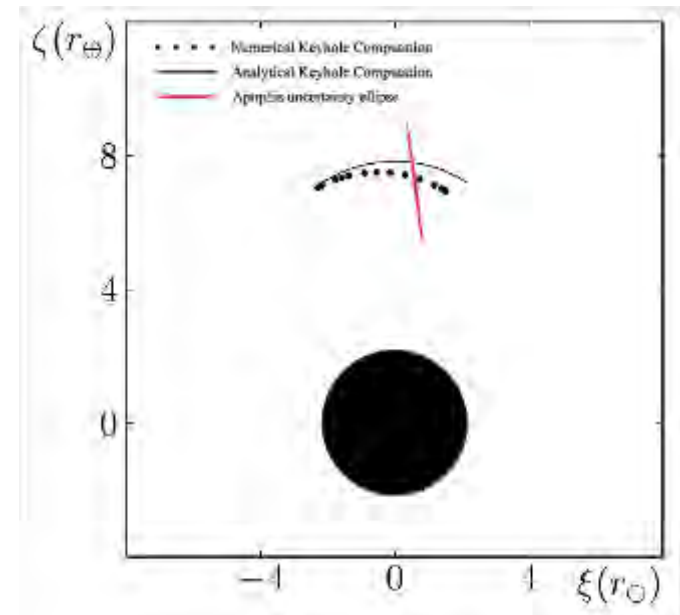
$$\Delta\zeta = \frac{3a}{\mu} V_E \sin\vartheta \int_0^{t_p} (t_s - \tau) \mathbf{v} \cdot \mathbf{A} d\tau$$

Valid for all non destructive deflection strategies!!

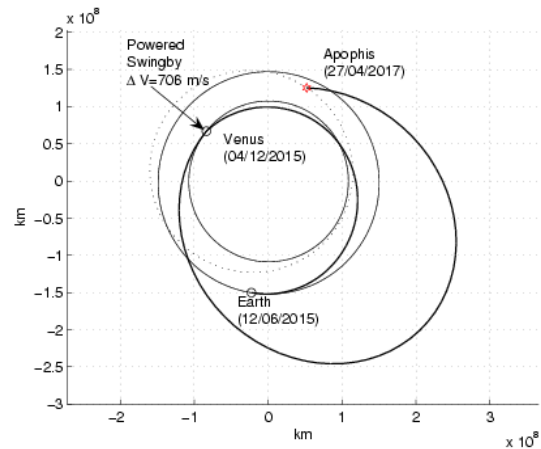
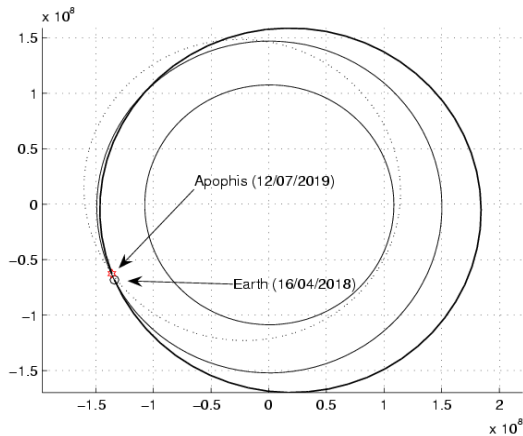




1. In late December 2004 impact monitoring robots found that NEA 2004 MN4 would have an extremely close approach with the Earth on the 13th of April 2029, with a probability of hitting our planet that rose up to 2.6% on the 27th of December.
2. In that date some previous observation data were mined from the Spacewatch archives and resulted in dropping significantly the impact risk.
3. Further observation from Arecibo, the 27th, 29th and 30th of January 2005 served to exclude the impact risk.



Apophis may teach us a lot on how to plan a real deflection mission



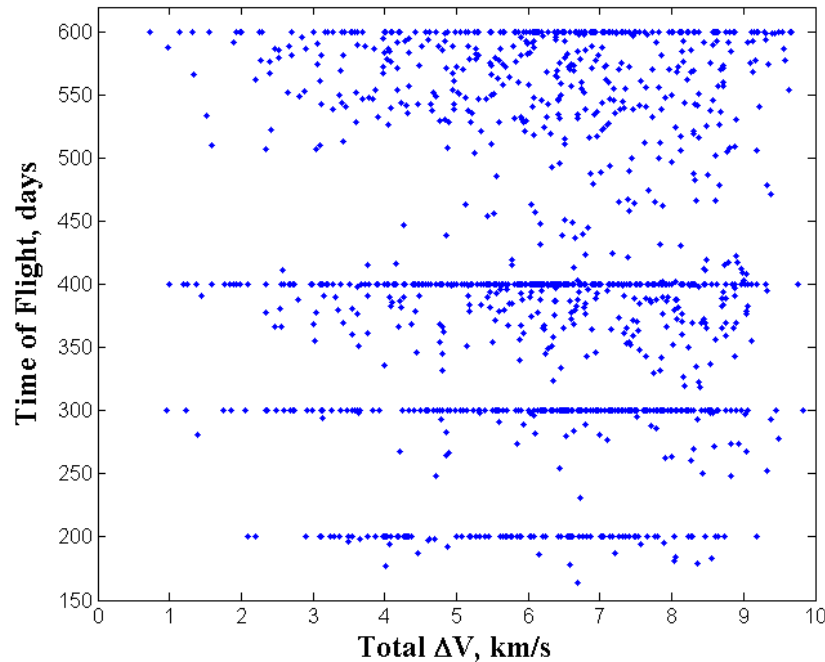
Departure	Impact	$ U $ km/sec	Deflection $\Delta\zeta$, km
16/4/2011	12/6/2012	7.93	87.8
25/4/2012	4/5/2013	8.14	63
25/9/2013	27/3/2014	8.29	43.5
3/6/2014	26/2/2015	10.9	18
16/4/2018	12/7/2019	7.32	57.58
17/4/2019	31/5/2020	8.02	43.37
29/12/2020	9/3/2021	7.15	47.4
8/9/2021	25/3/2022	9.01	17.23
2/3/2026	2/3/2027	7.09	10.05
1/4/2027	2/12/2027	7	3.96
5/2/2028	18/11/2028	5	0.45

Don Quijote spacecraft as designed by the Concurrent Design Facility at ESA (2004)

2006: Pre-keyhole deflection of Apophis is possible with DQ!!



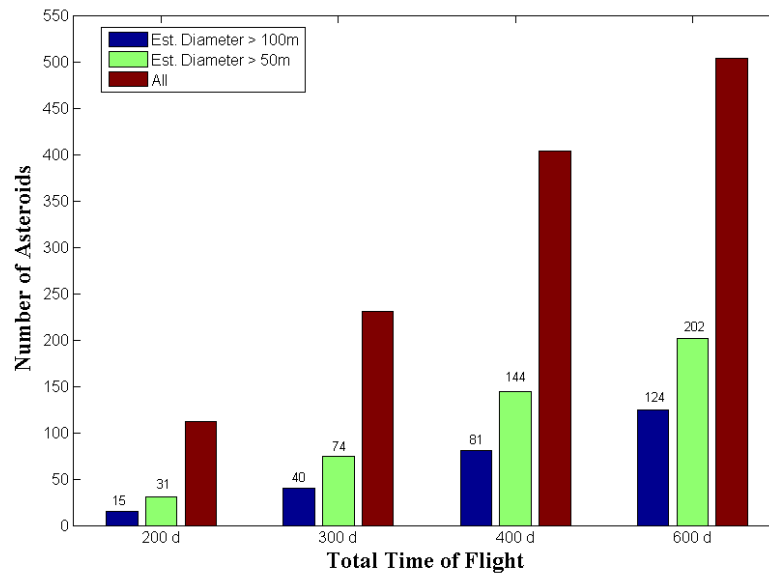
Human Missions to Asteroids



- Asteroid selection from the MPCORB database
- ~50.000 GO problems solved in 3 hours!!
- MGA-DSM model



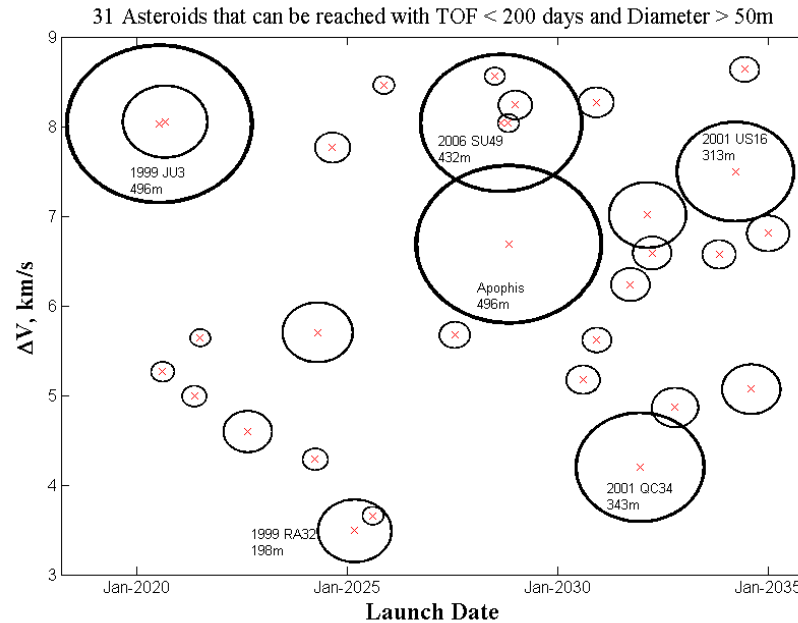
Obama's vision



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Obama's vision



- Asteroid selection from the MPCORB database
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Obama's vision