An overview on current methods for interplanetary trajectory optimization

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dario.izzo "at" esa.int
monitor, perform and foster research on advanced space systems, innovative concepts and working methods
The Advanced Concepts Team

Scientific domains with weak links to space

Techniques developed in basic scientific research

too immature for regular ESA programmes or projects

Topics on which ESA is expected to have a position

Fundamentals of interplanetary Trajectory optimisation
1. Interplanetary Trajectories
   a. Real Missions
   b. The GTOC problems
2. Direct vs. Indirect method
   a. The Optimal Control Problem
   b. Pontryagin Maximum Principle (indirect methods)
   c. Transcription Methods (direct methods)
3. A deeper look into direct methods
   a. Building Blocks (Lambert, Lagrange, Mivovitch)
   b. Chemical propulsion
   c. Low-thrust propulsion
4. Web resources
5. Examples: asteroid deflection, human missions to asteroids
Interplanetary Trajectories
a. Real Missions
Water in Enceladus?
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Fundamentals of interplanetary Trajectory optimisation
100% Mercury
Rubble piles exist!
Bepi Colombo (ESA)

Fundamentals of interplanetary Trajectory optimisation

Advanced Concepts Team
Dawn (NASA)

Fundamentals of interplanetary Trajectory optimisation
Interplanetary Trajectories

b. GTOC problems
1. GTOC: Global Trajectory Optimization Competition
2. Taking place every year (roughly)
3. Near-to-impossible interplanetary trajectory problem: complexity ensures a clear competition winner
4. Open to academia, industry and space agencies
5. Winners organize and define the following edition
6. Creating a formidable database of challenging problems and solution methods
7. Competition duration is, usually, one month
8. The problem is rigorously defined so that solutions can be ranked with respect to a quantitative objective value
9. Web resources

The America’s cup of rocket science
### GTOC 1: Save the Earth

<table>
<thead>
<tr>
<th>Team name</th>
<th>Value</th>
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<tbody>
<tr>
<td>1. Jet Propulsion Laboratory</td>
<td>1,850,000</td>
</tr>
<tr>
<td>2. Deimos Space</td>
<td>1,820,000</td>
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<td>3. GMV</td>
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Fundamentals of interplanetary Trajectory optimisation

Advanced Concepts Team
## GTOC2: Multiple Asteroid Rendezvous

### Team Rankings

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<th>Value</th>
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<tr>
<td><strong>Advanced Concepts Team (ESA)</strong></td>
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<td>Centre National d’Etudes Spatiales (CNES)</td>
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<td>GMV Aerospace and Defence</td>
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<td>German Aerospace Center (DLR)</td>
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### GTOC3: Multiple Sample Return

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<td>Thales Alenia Space</td>
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<td>Tsinghua University</td>
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### GTOC4: Asteroids Billiard

#### Team name

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<tr>
<td>University of Texas at Austin</td>
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<tr>
<td>University of Glasgow &amp; University of Strathclyde</td>
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<td>University of Trento</td>
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<td>University of Bremen, Politecnico Milano</td>
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# GTOC5: Penetrators

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<td>2. Politecnico di Torino, Universita’ di Roma</td>
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<td>3. Tsinghua University, Beijing</td>
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<tr>
<td>Georgia institute of Technology</td>
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<td>15, 5394</td>
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<tr>
<td>DLR, Institute of Space Systems</td>
<td>14, 5438</td>
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<td>12.2, 5472</td>
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<td>VEGA Deutschland</td>
<td>12, 4873</td>
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<tr>
<td>University of Strathclyde, Glasgow</td>
<td>12, 5241</td>
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</tbody>
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**Fundamentals of interplanetary Trajectory optimisation**

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**Advanced Concepts Team**
<table>
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<tr>
<th>Team name</th>
<th>Value</th>
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<tbody>
<tr>
<td>1. Politecnico di Torino, Universita’ di Roma</td>
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<td>University of Colorado, Boulder</td>
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<td>U. of Jena, Germany &amp; TU Delft</td>
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<tr>
<td>Beihang University, Beijing</td>
<td>83</td>
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Direct vs. Indirect methods

a. The optimal control problem
The fundamental problem: OCP

minimize: \( J(x(t), u(t)) = \int_{t_0}^{t_1} F(x(t), u(t)) dt \)
subject to: \( \dot{x}(t) = f(x(t), u(t)) \)
\( x(t_0) \in S_0 \)
\( x(t_1) \in S_1 \)
\( u \in U \)
Example: Keplerian dynamic

\[ x \to [r, v, m] \]

\[ \dot{x}(t) = f(x(t), u(t)) \to \begin{cases} 
\dot{r} = v \\
\dot{v} = -\frac{\mu}{r^3}r + c_1 \frac{u}{m} \hat{u} \\
\dot{m} = -c_2 u 
\end{cases} \]

\[ x(t_0) \in S_0 \to r(t_0) = r_0, \quad v(t_0) = v_0, \quad m(t_0) = m_0 \]

\[ x(t_1) \in S_1 \to r(t_1) = r_1, \quad v(t_1) = v_1 \]

\[ F(x(t), u(t)) = -u_1 \]

\[ u \in \mathcal{U} \to u_1 \in [0, 1], \quad |\hat{u}| = 1 \]
Direct vs. **Indirect** methods
b. Pontryagin Maximum Principle
The maximum principle

\[ \mathcal{H}(\lambda(t), x(t), u(t)) = \lambda(t) \cdot f(x(t), u(t)) - F(x(t), u(t)) \]

\[ \dot{x} = \frac{d\mathcal{H}}{d\lambda} \]

\[ \dot{\lambda} = -\frac{d\mathcal{H}}{dx} \]

(i) \[ \mathcal{H}(\lambda(t), x(t), u^*(t)) \geq \mathcal{H}(\lambda(t), x(t), u(t)), \quad \forall u(t) \in \mathcal{U} \]

(ii) \[ \mathcal{H}(\lambda(t_1), x(t_1), u^*(t_1)) = 0 \]
Example: Keplerian dynamic

\[ \mathcal{H}(\lambda(t), x(t), u(t)) = \lambda_r(t) \cdot v(t) + \lambda_v(t) \cdot \left( -\frac{\mu}{r^3} \mathbf{r} + c_1 \frac{u}{m} \mathbf{i}_u \right) - \lambda_m c_2 u - u \]

\[ \begin{align*}
\dot{x} &= \frac{d\mathcal{H}}{d\lambda} \\
\dot{\lambda} &= -\frac{d\mathcal{H}}{dx}
\end{align*} \]

\[ \begin{align*}
\dot{r} &= v \\
\dot{v} &= -\frac{\mu}{r^3} \mathbf{r} + c_1 \frac{u}{m} \mathbf{i}_u \\
\dot{m} &= -c_2 u \\
\dot{\lambda}_r &= \frac{\mu}{r^3} \lambda_v - \frac{3\mu r \cdot \lambda_v}{r^5} \mathbf{r} \\
\dot{\lambda}_v &= -\lambda_r \\
\dot{\lambda}_m &= -c_1 \frac{u}{m^2} \lambda_v \cdot \mathbf{i}_u
\end{align*} \]
Example: Keplerian dynamic

\[
\mathcal{H}(\lambda(t), x(t), u(t)) = \lambda_r(t) \cdot v(t) + \lambda_v(t) \cdot \left( -\frac{\mu}{r^3} \mathbf{r} + \frac{c_1}{m} \mathbf{i} \right) - \lambda_m c_2 u - u
\]

Let us apply Pontryagin maximum principle:

\[
\hat{i}_u = \frac{\lambda_v}{\lambda_v}
\]

\[
\rho = \frac{c_1}{m} \lambda_v - c_2 \lambda_m - 1 \quad \text{Switching function}
\]

\[
\begin{cases}
  u = 0 & \rho < 0 \\
  u = 1 & \rho > 0 \\
  u \in [0, 1] & \rho = 0
\end{cases}
\]
Example: Keplerian dynamic

\[
H(\lambda(t), x(t), u(t)) = \lambda_r(t) \cdot v(t) + \lambda_v(t) \cdot \left(-\frac{\mu}{r^3} r + \frac{c_1}{m} u \hat{i}_u\right) - \lambda_m c_2 u - u
\]

\[
\begin{align*}
\dot{r} &= v \\
\dot{v} &= \frac{\mu}{r^3} r + c_1 \frac{u}{m} \lambda_v \\
\dot{m} &= -c_2 u \\
\dot{\lambda}_r &= \frac{\mu}{r^3} \lambda_v - 3\mu \frac{r \cdot \lambda_v}{r^3} r \\
\dot{\lambda}_v &= -\lambda_r \\
\dot{\lambda}_m &= -c_1 \frac{u}{m^2} \lambda_v
\end{align*}
\]

\[
\rho = \frac{c_1}{m} \lambda_v - c_2 \lambda_m - 1
\]

The OCP becomes a Two Points Boundary Value Problem TPBVP: find the costate initial values so that the final states are achieved at the final time (plus final condition on the hamiltonian if time is free, plus transversality conditions)
Direct vs. Indirect methods

b. Transcription Methods
minimize: \[ J(x(t), u(t)) = \int_{t_0}^{t_1} F(x(t), u(t)) dt \]
subject to: \[ \dot{x}(t) = f(x(t), u(t)) \]
\[ x(t_0) \in S_0 \]
\[ x(t_1) \in S_1 \]
\[ u \in U \]

\[ \min J(y) \]
\[ \text{subject to: } g(y) \leq 0 \]
Summary

Indirect methods: OCP -> TPBVP

Direct methods: OCP -> NLP
How difficult can it be?

\[
\begin{align*}
\text{find: } & \quad x \in \mathbb{R}^n \times \mathbb{N}^m \\
\text{to minimize: } & \quad f(x) : \mathbb{R}^n \times \mathbb{N}^m \to \mathbb{R}^p \\
\text{subject to: } & \quad g(x) \leq 0
\end{align*}
\]
Building blocks
ii. Lagrange propagation
1. Predicting the time evolution of an orbit from starting conditions
2. It is an initial value problem (Cauchy)
3. Its solution can be efficiently obtained in terms of the Lagrange coefficients $F,G$
4. Kepler’s equation needs to be solved to invert the eccentric anomaly - time relation.

\[
\begin{align*}
\dot{r} &= v \\
\dot{v} &= -\frac{\mu}{r^3} r \\
\mathbf{r}(0) &= \mathbf{r}_0 \\
\mathbf{v}(0) &= \mathbf{v}_0
\end{align*}
\]
Building blocks
i. Lambert’s Problem
1. Going from one point to another in a fixed time.
2. It is, again, a TPBVP.
3. Its modern solution relies on results from Lambert, Gauss, Lagrange and in more modern times Battin, Lancaster and Blanchard.
4. It turns out that all Lambert problems have 1 solution and, according to the transfer time, may also have $2N$ multiple revolution solutions.

\[
\begin{align*}
\dot{r} &= v \\
\dot{v} &= -\frac{\mu}{r^3}r \\
\mathbf{r}(0) &= \mathbf{r}_1 \\
\mathbf{r}(T) &= \mathbf{r}_2
\end{align*}
\]
Lambert’s Problem
Building blocks

iii. Mivovitch Fly-bys
Mivovitch sling-shot

Fly by $\rightarrow$ rotation of relative velocity
Mivovitch sling-shot

\[ \mathbf{v}_{sc}^- \]
\[ \mathbf{v}_{sc}^+ \]
\[ \mathbf{v}_\infty^- \]
\[ \mathbf{v}_\infty^+ \]
\[ \mathbf{v}_{pla} \]
Spacecraft velocity has changed in the absolute frame

Watch Mivovitch Video
A deeper look into direct methods

i. Chemical propulsion
Given a planetary sequence of $N$ planets find:

$$x = [t_{dep}, T_1, \ldots, T_N, T_{arr}]$$

To minimise:

$$J = \Delta V_1 + \Delta V_2 + \ldots + \Delta V_{arr} \ (\Delta V_{dep})$$

Subject to:

$$x \in [x, \bar{x}] \quad \text{Launch window constraint}$$

$$\Delta V_{dep}^2 < C_3^{\text{launch}} \quad \text{Launcher constraint}$$

Planetary sequence: EVVEVMe

MGA: Model
1. Earth-Mars transfer
2. Chemical propulsion
3. 200 days of transfer
4. MJD2000 used
1. Earth-Mars transfer
2. Chemical propulsion
3. Days and MJD2000 used
1. Earth-Jupiter-Saturn transfer
2. Local optima cluster together
3. Better local optima are close to the global one
4. Clustered local optima have similar objective values
MGA-1DSM: model

\[ x = [t_0, V_\infty, u, v, \eta, T_0] + \ldots + [r_p, \beta, \eta_i, T_i] \]

- Features of the MGA-1DSM model:
  - DSM value can be zero
  - Multi-revs are included
  - Resonant returns and backflips included
- Multiple objectives can be handled
  (http://sophia.estec.esa.int/gtocwiki/index.php/Final_Solution)
MGA-1DSM: cluster pruning

Developed as part of the work for the joint JPL/ESA Outer Planets Mission Analysis Working Group (2007-2008)
### MGA-1DSM: The TandEM case

**Spacecraft**

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<tr>
<td>Arrival mass</td>
<td>1476.03 kg</td>
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<tr>
<td>$I_{sp}$</td>
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**Departure**

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<tr>
<td>$V_{\infty}$</td>
<td>3.34 km/s</td>
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<tr>
<td>Declination</td>
<td>3.1 deg</td>
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**Cruise**

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<td>Earth fly-by</td>
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<td>DSM Epoch</td>
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<td>DSM $\Delta V$</td>
<td>167 m/s</td>
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<td>Earth fly-by</td>
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**Arrival**

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<tr>
<td>$V_{\infty}$</td>
<td>0.676 km/s</td>
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<tr>
<td>Total flight time</td>
<td>9.63 years</td>
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MGA-LT: model

$t_0$
MGA-LT: model

$t_0, \mathbf{v}_\infty$
$t_0, v_\infty, \Delta V_1$
MGA-LT: model

\( t_0, v_\infty, \Delta V_1 \)
MGA-LT: model

\[ t_0, v_\infty, \Delta V_1, \Delta V_2 \]
MGA-LT: model

$t_0, v_\infty, \Delta V_1, \Delta V_2$
$x = [t_0]$

$+ [T_1, m_{f1}, V_{x1}, V_{y1}, V_{z1}, V_{xf1}, V_{yf1}, V_{zf1}]$

$+ [T_2, m_{f2}, V_{x2}, V_{y2}, V_{z2}, V_{xf2}, V_{yf2}, V_{zf2}] + ...$

$+ [u_1^x, u_1^y, u_1^z] + [u_2^x, u_2^y, u_2^z] + ...$

**Constraints:**

**Mismatch**

$|u_i|$

$|V_{f_i}| = |V_{f_{i+1}}|$

$V_{f_i} \cdot V_{f_{i+1}} \geq \alpha$

- Features of the MGA-LT model:
  - Easy switch between low and high fidelity
  - Large convergence radius
On line resources
GTOP - A Trajectory Problem Database
Crowdsourcing Experiments (html5)
The Space Game
Jupiter Hopper
Advanced Concepts Team
Fundamentals of interplanetary Trajectory optimisation

SOCIS
PyKEP
PyGMO
Asteroid deflection: What the dinosaurs did not know
Asteroid deflection: What the dinosaurs did not know!

- Impacts happen!!!
- From mass extinctions to regional to local damage
- We know of ~500,000 asteroids in the solar system
- ~800 are potentially hazardous
- What the dinosaurs did not know (video)

Tunguska 1908 (icy comet? Small asteroid?)

Winslow, Arizona (~49,000 years ago)
Past impacts

Map of confirmed (red), perspective for verification (magenta) and proposed for further study (blue) impact structures on the Earth. Size of circles is proportional to the crater diameter.

Source: Institute of Computational Mathematics and Mathematical Geophysics. Novosibirsk, RUSSIA
ESA’s Don Quijote Mission

- 2003: 6 industrial studies on the topic of NEOs characterization and hazard mitigation
- 2004: DQ is recommended as highest priority for near-term implementation of NEO missions (NEOMAP): “Don Quijote has the potential to teach us a great deal, not only about the internal structure of a NEO, but also about how to mechanically interact with it”
- First time that the deflection of a celestial body may be proved and studied

The orbital mechanics of moving asteroids needed to be studied in more details
A dart game?
The asteroid deflection formula

\[ \Delta \zeta = \frac{3a}{\mu} V_E \sin \theta \int_{0}^{t_p} (t_s - \tau) \mathbf{v} \cdot \mathbf{A} \, d\tau \]

Valid for all non destructive deflection strategies!!
Deflection charts

Apophis (encounter 2029)

200 km
600 km
1000 km
1200 km

\( \tau_p \)

9000 km
7000 km
5000 km
3000 km
1000 km

\( \tilde{z} \)

2007 VK184 (encounter in 2044)
1. In late December 2004 impact monitoring robots found that NEA 2004 MN4 would have an extremely close approach with the Earth on the 13th of April 2029, with a probability of hitting our planet that rose up to 2.6% on the 27th of December.

2. In that date some previous observation data were mined from the Spacewatch archives and resulted in dropping significantly the impact risk.

3. Further observation from Arecibo, the 27th, 29th and 30th of January 2005 served to exclude the impact risk.

Apophis may teach us a lot on how to plan a real deflection mission.
Globally optimal trajectories

Don Quijote spacecraft as designed by the Concurrent Design Facility at ESA (2004)

2006: Pre-keyhole deflection of Apophis is possible with DQ!!
Human Missions to Asteroids
MR-DSM: Human Mission to Asteroids in 3 hours!!

- Asteroid selection from the MPCORB database
- ~50,000 GO problems solved in 3 hours!!
- MGA-DSM model
- Asteroid selection from the MPCORB database
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Advanced Concepts Team

Fundamentals of interplanetary Trajectory optimisation

- Asteroid selection from the MPCORB database
- ~50,000 GO problems solved in 3 hours!!
- MGA-DSM model

MR-DSM: Human Mission to Asteroids in 3 hours!!